17th INTERNATIONAL WORKSHOP ON REAL & COMPLEX SINGULARITIES

July 24 - 29, 2022

ICMC, University of São Paulo
São Carlos, Brazil

*Special sessions to honor **Osamu Saeki's** work and his 60th birthday.*

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realization and support

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Introduction

This is a biennial conference realized by the Singularity Theory Group, São Carlos, Brazil. It has been established as one of the key events on singularity theory, algebraic geometry, bifurcation theory and related areas. It brings together internationally renowned and young researchers to report on their recent achievements, to exchange ideas and to address trends of research in a highly stimulating environment.

In 2022, São Carlos holds the 17th edition of the Workshop on Real and Complex Singularities, at the Instituto de Ciências Matemáticas e de Computação, University of São Paulo, Brazil. This edition marks the return to holding in-person events after the COVID-19 pandemic.

We have the honor of organizing this conference celebrating Professor Osamu Saeki's works and his 60th birthday.

We appreciate the suport of: ICMC/USP, UFSCar, CAPES, CNPq, FAPESP, INCT-Mat and IMU.

Contents

1. Plenary Talks

On the Fukui-Kurdyka-Paunescu conjecture

Alexandre Cesar Gurgel fernandes (Federal University of Ceara/Brazil)

Abstract: In this talk, we will address the conjecture of Fukui-Kurdyka-Paunescu, which says that subanalytic arc-analytic bi-Lipschitz homeomorphisms preserve the multiplicities of real analytic sets.

This is a joint work with Edson Sampaio and Zbigniew Jelonek.

Polar exploration of complex surface germs

Anne PICHON (Aix Marseille University/France)

Abstract: A normal complex surface singularity $(X, 0)$ can be resolved either by a sequence of normalized point blowups, following seminal work of Zariski from the late nineteen thirties, or by a sequence of normalized Nash transforms, as was done half a century later by Spivakovsky. The main goal of this talk is to shed some light on the relationship between these two resolution algorithms, which despite their importance and their centrality in modern mathematics is still quite mysterious, providing some evidence of a duality between the two which was initially observed by Lê D.T. While the blowup $\mathrm{Bl}_0 X$ of the maximal ideal of $(X, 0)$ is the minimal transformation which resolves the family of generic hyperplane sections of $(X, 0)$, the Nash transform ν of $(X, 0)$ is the minimal transformation that resolves the family of the polar curves associated with the generic plane projections of $(X, 0)$. Therefore, the study of the duality of resolution algorithms translates into the study of the relative positions on $(X, 0)$ of those two families of curves. This is the viewpoint we will adopt in this talk. Our main theorem roughly states that fixed the topology of $(X, 0)$, that is the homeomorphism class of its link, there are, up to homeomorphism, only a finite number of possible relative positions between these families of curves. This fits in the program of *polar explorations*, the quest to determine the generic polar variety of a singular surface germ, to which part of the talk will be devoted. This is joint work with André Belotto da Silva, Lorenzo Fantini, András Némethi [1].

References

[1] André Belotto da Silva, Lorenzo Fantini, András Némethi, Polar exploration of complex surface germs, (2021), arXiv:2103.15444, to appear in *Transactions of the American Mathematical Society.*

Zeta-function and μ^*-Zariski pairs of surfaces

Christophe Eyral (Institute of Mathematics, Polish Academy of Siences/Poland)

Abstract: A Zariski pair of surfaces is a pair of complex polynomial functions in $\mathbb{C}^3$ which is obtained from a classical Zariski pair of projective curves $f_0(z_1, z_2, z_3) = 0$ and $f_1(z_1, z_2, z_3) = 0$ of degree d in $\mathbb{P}^2$ by adding a same term of the form z_i^{d+m} $(m \geq 1)$ to both f_0 and f_1 so that the corresponding affine surfaces of $\mathbb{C}^3$ — defined by $g_0 := f_0 + z_i^{d+m}$ and $g_1 := f_1 + z_i^{d+m}$ — have an isolated singularity at the origin and the same zeta-function for the monodromy associated with their Milnor fibrations (so, in particular, g_0 and g_1 have the same Milnor number). In this talk, I will show that if f_0 and f_1 are "convenient" with respect to the coordinates (z_1, z_2, z_3) and if the singularities of the curves $f_0 = 0$ and $f_1 = 0$ are Newton non-degenerate in some suitable local coordinates, then (g_0, g_1) is a μ^*-Zariski pair of surfaces, that is, a Zariski pair of surfaces whose polynomials g_0 and g_1 have the same Teissier's μ^*-sequence but lie in different path-connected components of the μ^*-constant stratum. The proof is based on a new general formula which gives, under appropriate conditions, the Milnor number of functions of the above type, together with the observation that two polynomials functions lying in the same path-connected component of the μ^*-constant stratum can always be joined by a "piecewise complex-analytic path". This is a joint work with Mutsuo Oka.

Intersection Spaces, Mirror Symmetry and Toric Varieties

Jan Timo Essig (Kiel/Germany)

Abstract: Intersection Spaces are a spatial approach to Poincaré Duality on singular stratified pseudomanifolds. The concept is different from intersection homology. In fact, for conifold transitions, both concepts are related by mirror symmetry. In the talk, we move from isolated singularities via a depth one setting with flat fiber bundles to toric varieties. We examine the spatial as well as the differential form and sheaf theoretic approach.

The butterfly umbilic points on holomorphic plane curves

Jorge Luiz Deolindo Silva (Universidade Federal de Santa Catarina/Brazil)

Abstract: We consider the geometry of regular holomorphic curves in $\mathbb{C}^2$ viewed as surfaces in the affine space $\mathbb{R}^4$. We study the $\mathcal{A}$-singularities of parallel projections of generic such surfaces along planes to transverse planes. We show that, at any point on the surface which is not an inflection point of the curve, there are two tangent directions determining two planes along which the projection have singularities of type butterfly or worse. The integral curves of these directions form a pair of foliations on the surface defined by a binary differential equation (BDE). The

singularities of this BDE are the inflection points of the curve together with other points that we call *butterfly umbilic points*. We determine the configurations of the solution curves of the BDE at its singularities. Finally, we prove that an affine view of an algebraic curve of degree $d \geq 2$ in $\mathbb{C}P^2$ has $8d(d-2)$ butterfly umbilic points.

The problems of resolution of singularities and local uniformization in arbitrary characteritic

Mark Spivakovsky (Paul Sabatier/France)

Abstract: The problem of resolution of singularities asks whether, given an algebraic variety X over a field, there exists a non-singular algebraic variety X' and a proper map $X' \to X$ which is one-to-one over the non-singular locus of X. If we cover X' by affine charts, the problem becomes one of *parametrizing* pieces of X by small pieces of the Euclidean space k^n. This local version of the problem, called Local Uniformization, is stated in terms of valuations, as follows. Let (R, M, k) be a local noetherian domain and R_ν a valuation ring containing R and having the same field of fractions as R. The **Local Uniformization Theorem** asserts the existence a *regular* finite type R-algebra R' such that $R' \subset R_\nu$; it was proved by O. Zariski in 1940 in the case when *char* $k = 0$ and is one of the central open problems in the field when *char* $k = p > 0$. We will start out by recalling Zariski's valuative approach to the problem as well as Hironaka's (non-valuative) proof of resolution in characteristic zero. We will explain the difficulties arising in characteristic $p > 0$ and the idea for overcoming them using differential operators. At the end of the talk we will discuss two main technical tools of our program: key polynomials and universal Puiseux epanstion in generalized power series with non-well-ordered support.

Poincare-Reeb graphs of real algebraic domains

Miruna-Stefana Sorea (SISSA/Italy)

Abstract: Consider a real bivariate polynomial function that has a strict local minimum at the origin and that vanishes at this point. In a sufficiently small neighborhood of the origin, the non-zero level curves of this function are smooth Jordan curves. Whenever the origin is a Morse strict local minimum, the small enough level curves become boundaries of convex topological disks. Otherwise, the levels may be non-convex, as it was proven by M. Coste. In order to measure this non-convexity, we introduce a combinatorial object called the Poincaré-Reeb tree associated to a level curve and to a projection direction. Our goal is to characterize all topological types of Poincaré-Reeb trees. I will explain how to construct a family of polynomials that realizes a large class of these trees. Moreover, in a joint work with Arnaud Bodin and Patrick Popescu-Pampu, we extend the previous method of study of non-convexity to real algebraic domains.

Milnor number of nondegenerate isolated complete intersection singularities

Nguyen Tat Thang (Hanoi Institute of Mathematics/Vietnam)

Abstract: We prove that for two germs of analytic mappings $f, g : (C^n, 0) \to (C^p, 0)$ with the same Newton polyhedra which are (Khovanskii) non-degenerate and their zero sets are complete intersections with isolated singularity at the origin, there is a piecewise analytic family ft of analytic maps with f0 = f, f1 = g which has a so-called uniform stable radius for the Milnor fibration. As a corollary, we show that their Milnor numbers are equal. Also, a formula for the Milnor number is given in terms of the Newton polyhedra of the component functions. This is a generalization of the result by C. Bivia-Ausina.

On invariants of a quasi-homogeneous map germ

Otoniel Nogueira da Silva (Universidade Federal da Paraíba/Brazil)

Abstract: In this talk, we consider a quasi-homogeneous, corank 1, finitely determined map germ f from $(\mathbb{C}^2, 0)$ to $(\mathbb{C}^3, 0)$. We consider the invariants $m(f(D(f))$ and J, where $m(f(D(f))$ denotes the multiplicity of the image of the double point curve $D(f)$ of f and J denotes the number of tacnodes that appears in a stabilization of the transversal slice curve of $f(\mathbb{C}^2)$. We present formulas to calculate $m(f(D(f))$ and J in terms of the weights and degrees of f. We also describe the embedded topological type of a generic hyperplane section of $f(\mathbb{C}^2)$, denoted by γ_f, in terms of the weights and degrees of f. As a consequence, a necessary condition for a corank 1 finitely determined map germ $g : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$ to be quasi-homogeneous is that the plane curve γ_g has either two or three characteristic exponents. As an application of our main result, we also show that any one-parameter unfolding $F = (f_t, t)$ of f which adds only terms of the same degrees as the degrees of f is Whitney equisingular.

Milnor number vs Tjurina number of isolated hypersurface singularities

Patricio Almirón Cuadros (Universidad Complutense de Madrid/Spain)

Abstract: The Milnor and Tjurina numbers are two of the most important invariants of an isolated hypersurface singularity. While being of different nature, the Milnor number is a topological invariant and the Tjurina number is an analytic invariant, their quotient, or equivalently their difference, provides a measure of how far is our singularity to be quasi-homogeneous. In this talk we will adress the problem of finding sharp upper bounds for the quotient of the Milnor and

Tjurina numbers. The first part of the talk will be devoted to introduce the motivations to study the problem of finding sharp upper bounds for the quotient of the Milnor and Tjurina numbers. We will show particular solutions in the case of plane curve singularities and surface singularities which will link our problem with other conjectures in singularity theory. In the second part of the talk, we will focus on the comparison of the Milnor and Tjurina numbers in the case of a Sebastiani-Thom type singularity. We will show the different behaviour of those invariants in that case and we will show some upper bounds for their quotient.

Four-manifolds via singular fibrations

Refik Inanç Baykur (University of Massachusetts Amherst/United States of America)

Abstract: We will discuss how the existence of certain geometric and exotic smooth structures on four-manifolds can be effectively studied using surface fibrations with the simplest types of real and complex singularities.

Towards Mond's conjecture

Roberto Giménez Conejero (Renyi institute/Hungary)

Abstract: Isolated singularities of hypersurfaces are similar, in many ways, to isolated instabilities of germs of functions. In this talk we will deal with germs $f : (\mathbb{C}^n, S) \to (\mathbb{C}^p, 0)$ where $p = n + 1$ (and also $n < p$).

An example of these similarities is that there are analogues of the Milnor number μ and Tjurina number τ, invariants that control the homotopy of a generic fiber and the *set* of possible perturbations (respectively). The analogues for map germs of these invariants are the image Milnor number μ_I and the $\mathcal{A}_e$-codimension (respectively).

There is, however, a question that keeps appearing when one is working with these map germs. While the relation between the Milnor and Tjurina numbers is an undergraduate exercise, i.e., $\mu \geq \tau$, with equality in the quasi-homogeneous case, the same relation is an open problem (in general, e.g. $n > 2$) that seems completely reasonable to believe true.

This is called Mond's conjecture and in this talk we present some recent results related to these invariants, as well as prove the conjecture in some cases. If there is enough time, we will also suggest some unexplored problems in this setting.

The results about the $\mathcal{A}_e$-codimension are part of a joint collaboration with Juan José Nuño-Ballesteros.

Determinantal Singularities, Newton polyhedra, and Euler obstruction

Thaís Maria Dalbelo (Federal University of São Carlos/Brazil)

Abstract: Using Esterov's results [1] and equivalence between matrices we define the Newton polyhedron of a matrix. We also present a formula for the local Euler obstruction of isolated determinantal singularities in terms of volumes associated to Newton polyhedra.

Joint work with Luiz Hartmann and Maicom Varella.

References

[1] Esterov, A. *Determinantal singularities and Newton polyhedra*, Tr. Mat. Inst. Steklova, 20–38 (2007).

Equimultiplicity of μ-constant families

Tomasz Ryszard Pełka (BCAM/Spain)

Abstract: I will present my recent joint work with J. F. de Bobadilla, proving that a family of isolated hypersurface singularities with constant Milnor number has constant multiplicity. The key idea is to endow the A'Campo model of "radius zero" monodromy with a symplectic structure. This new approach allows to generalize a spectral sequence of McLean converging to fixed point Floer Homology of iterates of the monodromy to a more general setting, which is well suited to study μ-constant families. If time permits, I will outline possible applications of the "radius zero" construction to other degeneration problems.

2. Special talks

From Seki to Saeki

Jean-Paul Brasselet (Aix-Marseille Université/France)

Abstract: Mathematics has evolved impressively in Japan from the days of Seki Kowa (1640-1708) to that of Saeki Osamu. A little insight into Japanese mathematics.

Topology of Reeb spaces of smooth functions on manifolds

Osamu Saeki (Kyushu University/Japan)

Abstract: The Reeb space of a continuous function is the space of connected components of the level sets. In this talk, we first prove that the Reeb space of a smooth function on a closed manifold with finitely many critical values has the structure of a finite graph without loops. We also study a similar space defined by path-components instead of usual components. We will also try to give various interesting examples.

3. Special talks in honor of Prof. Saeki

Cobordism theory of Morse functions and applications

Dominik Johannes Wrazidlo (Heidelberg University/Germany)

Abstract: Cobordism groups of various types of Morse functions have been studied by several authors by applying explicit methods of global singularity theory of differentiable maps. For instance, Ikegami used Levine's cusp elimination technique to compute cobordism groups of Morse functions on closed manifolds (this generalized results of Ikegami-Saeki and Kalmàr). An application of Ikegami's techniques to the construction of topological invariants of generic differentiable map germs was found by Ikegami and Saeki. Saeki and Yamamoto studied Morse functions on compact surfaces with boundary up to so-called admissible cobordism by using the cohomology of the universal complex of singular fibers, as well as a combinatorial argument based on labeled Reeb graphs. By using similar techniques, Yamamoto studied versions of these cobordism groups without cusps. Saeki applied the technique of Stein factorization and Cerf's pseudoisotopy theorem to study cobordism groups of so-called special generic functions, i.e., Morse functions with only maxima and minima as their critical points. In this talk, we impose more general index constraints on the Morse functions, and study the resulting cobordism relations for such "constrained" Morse functions by means of the two-index theorem of Hatcher and Wagoner, as well as handle extension techniques for fold maps due to Gay and Kirby. As an application to high-dimensional topological field theory, we show how exotic Kervaire spheres can be distinguished in infinitely many dimensions from other exotic spheres as elements of the cobordism group of constrained Morse functions.

Singularity theory for mappings with no singularity

Masamichi Takase (Seikei University/Japan)

Abstract: I would like to talk about past research which were done in collaboration with Professor Osamu Saeki or heavily used his previous results. Most of them are on smooth embeddings or immersions (i.e. mappings with no singularity) of manifolds; nevertheless, the results or the proofs are given in terms of singularities of related smooth mappings.

The theory of singular fibers of differentiable maps and its applications

Takahiro Yamamoto (Tokyo Gakugei University)

Abstract: One of Osamu Saeki' s contributions in singularity theory is developing the theory of singular fibers of differentiable maps. In this talk, we give an overview of the theory of singular fibers of differentiable maps and its applications.

Global Theory of Singularities of Differentiable Maps – Osamu Saeki's works and others

Toru Ohmoto (Hokkaido University/Japan)

Abstract: Special session is given by four presenters in different generations (30-, 40-, 50-, nearly 60). As the first speaker, and as an old friend over 30 years, I will give a very brief outline of Osamu's mathematical achievements, especially focusing on 4-dimensional topology via stable maps to lower dimensioal spaces.

4. Parallel Talks

Some Results in Combinatorics and Stabilizers

Alberto León Kushner Schnur (Facultad de Ciéncias - UNAM/Mexico)

Abstract: In this talk we give some interesting equalities in combinatorics. Afterwards we compute elements of stabilizers for some particular cases in five and seven variables of degree p, for a prime p. We also study the general case for diagonal matrices with $2l$ entries equal to i, $2l-1$ entries equal to i, and one equal to $-i$, $\cdots$, finally $2l$ entries equal to $-i$. We propose a formula relating the cases $2l$ and $2l+2$ with the difference a quadratic in l, and give the cases with six and four, and eight and six variables.

The relative Bruce-Roberts numbers of a function on an isolated complete intersection singularity

Bárbara Karoline de Lima Pereira (UFSCar/Brazil)

Abstract: In this talk I will give the formula for the relative Bruce-Roberts number of a function f with respect to an ICIS $(X,0)$, $\mu^{-}_{BR}(f,X)$. In this work we prove that

$$\mu^{-}_{BR}(f,X) = \mu(f^{-1}(0)\cap X,0) + \mu(X,0) - \tau(X,0),$$

where μ and τ are the Milnor and Tjurina numbers, respectively, of the ICIS. We also consider the relative logarithmic characteristic variety, $LC(X)^{-}$, and we show that $LC(X)^{-}$ is Cohen-Macaulay. This is a joint work with J. J. Nuño-Ballesteros (Universitat de Valencia, SPAIN), B. Oréfice-Okamoto, (UFSCar, BRAZIL) and J.N. Tomazella, (UFSCar, BRAZIL).

Real algebraic links, braids and semiholomorphic polynomials

Benjamin Bode (Instituto de Ciencias Matemáticas (ICMAT)/Spain)

Abstract: A link in the 3-sphere is called *real algebraic* if it arises as the link of an isolated singularity of a polynomial map $f : \mathbb{R}^4 \to \mathbb{R}^2$. The set of real algebraic links is conjectured to be equal to the set of fibered links, but (in contrast to the algebraic links) this family is not characterised yet. In this talk I will discuss constructions of real algebraic links and their relation to braids and semiholomorphic polynomials, which are mixed functions that are holomorphic with respect to one complex variable. In particular, I show that for every homogeneous braid B the closure of B^2 is real algebraic. This implies that every link is a sublink of a real algebraic link.

Singularities of Frontal Surfaces

Christian Muñoz Cabello (University of Valencia/Spain)

Abstract: A hypersurface $X \subset \mathbb{C}^n$ is frontal if it admits a smooth unit normal vector field ν on X. Frontal hypersurfaces emerge in many different areas of mathematics and physics, such as wave propagation, dynamic systems theory and differential geometry. In this joint work with J.J. Nuño-Ballesteros and R. Oset Sinha, we define a notion of frontal codimension and study and give a characterisation of surfaces with finite frontal codimension in terms of its double point space. We also give a frontal version of the Marar-Mond formulas, introduce the notion of frontal Milnor number and propose the equivalent to Mond's conjecture for frontal hypersurfaces.

Principal curvature lines near a partially umbilic point of codimension one

Débora Lopes da Silva (Umiversidade Federal de Sergipe/Brazil)

Abstract: In this work we study the mutually orthogonal foliations, in oriented three dimensional manifolds M^3, whose leaves are the integral curves of the principal curvature direction fields associated to immersions $\alpha : M^3 \to \mathbb{R}^4$. We describe these foliations around their singularities, which occur at points, called partially umbilic, where at least two principal curvatures coincide. Here we extend the contributions of R. Garcia, 1989, 2001, further elaborated by R. Garcia, D. Lopes and J. Sotomayor, 2015, concerning the study of the generic singularity patterns denominated D_1, D_2, D_3, D_{12} and D_{23}. To this end here we establish the principal configurations in a neighborhood of a partially umbilic point D^1_{13} which appear generically in one parameter families of hypersurfaces of $\mathbb{R}^4$.

Characteristic-free approach to Unfoldings (and further results)

Dmitry Kerner (Ben Gurion University/Israel)

Abstract: The classical Singularity Theory was done for real/complex-analytic (or C^r) function-germs. The essential tool in the study of (right/contact/left-right) equivalences was integration of vector fields. I will present the (purely algebraic) characteristic-free approach to the study of right/contact, and most importantly left-right equivalences. First go the determinacy questions, of the type "How large is the group orbit of a given germ?" Then goes the theory of unfoldings (triviality, versality, stability). Time-permitting I will present results of Mather-Yau/Hauser-Gaffney type and some results on stable maps.

Topology of mixed polynomials with a Newton non-degenerate boundary

Eder Leandro Sanchez Quiceno (USP/SC/Brazil)

Abstract: Let $f : \mathbb{C}^2 \to \mathbb{C}$ be a mixed polynomial, i.e., a complex polynomial of variables $(u, \bar{u}, v, \bar{v})$. If $f(0) = 0$ and $\Sigma_f \cap f^{-1}(0) = \{0\}$ as a set germ, where Σ_f denotes the singular set of f, then associated with f we have the well-defined link L_f, which is the intersection of the mixed hypersurface $V_f := f^{-1}(0)$ with a 3-sphere of small radius centered at the origin.

In this talk we introduce the non-degenerate condition of the Newton boundary of a mixed polynomial. It allows us to get the condition $\Sigma_f \cap f^{-1}(0) = \{0\}$ and also a topological description of the link L_f from topological data obtained from the Newton boundary.

Joint work with: Raimundo Nonato Araújo dos Santos and Benjamin Bode.

Acknowledgements: The research was supported by grant 2017/25902-8, São Paulo Research Foundation (FAPESP) and was partially supported by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES).

Holomorphic foliations admitting sectorial first integrals

Fernando Pereira Paulucio Reis (Universidade Federal do Espírito Santo/Brazil)

Abstract: In this work, we study the interplay between the dynamics of a holomorphic vector field near a singular point in dimension two and the behavior of its orbits in some sectors obtained from a transverse section to a separatrix. More precisely, we address the question: Under wich circumstances the existence of a first integral in some sector assures the existence of a first integral in a neighborhood of the singularity. We addres this problem by combining some holomorphic foliations holonomy techniques together with some classical one variable asymptotic expansion techmiques.

Joint work with: Bruno Scardua (UFRJ).

Milnor set of real polynomials and detection of atypical values

Gabriel Esteban Perico Monsalve (Universidade de São Paulo/Brazil)

Abstract: Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a polynomial function. The Milnor set at infinity of f is defined as the set of points where the fibers of f are tangent to large enough spheres in $\mathbb{R}^2$. In this talk

we use properties of the Milnor set in order to detect the phenomena of vanishing and splitting at infinity which produce bifurcation values.

Acknowledgements: We would like to thanks grant #2020/14111-2 São Paulo Research Foundation (FAPESP) by financial support.

References

[1] M. Coste, M. J. de la Puente, *Atypical values at infinity of a polynomial function on the real plane: an erratum, and an algorithmic criterion*, J. Pure Appl. Algebra **162** (2001), no. 1, 23-35.

Applications of Singularity Theory in Thermodynamics

Graham Mark Reeve (Liverpool Hope University/United Kingdom)

Abstract: In this project I shall apply techniques from singularity theory to problems in Thermodynamics that were first considered in the 19th century by D.J. Korteweg. For a mixture of fluids, the so called coexistence curve on the corresponding isothermal Helmholtz free energy surface is the locus of points for which the pressure and chemical potential are the same in coexisting phases. These coexistence curves are related to purely geometric objects called Centre Symmetry Sets (CSS) and these occur where the criminant component of the CSS is tangent the surface. Much is known about these coexistence curves from physical and numerical experiments since the 19th century, but here I provide a complete classification of the normal forms, and hence the possible behaviour of these coexistence curves (up to diffeomorphism) for 1-parameter families of (Helmoltz free energy) surfaces. This is joint work Peter Giblin.

New development in Lipschitz Normally Embeded Singularities

Helge Moeller Pedersen (UFC/Brazil)

Abstract: Any real or complex singularity $(X, 0)$ is equipped with two natural metrics. The *outer metric*, which is the restriction of the ambient euclidean metric, and the *inner metric*, which is the metric associated with a riemannian metric on the germ. Up to bilipschitz equivalence these metrics does not depends on the choices of analytic embedding. The inner and outer metrics are in general not bilipschitz equivalent, and one says that $(X, 0)$ is *Lipschitz normally embedded* if they are. In this talk we will give an overview of the subject and discuss the current state of the question about which singularities are Lipschitz normally embedded. Including work still in progress joint with Fantini, Pichon and Schober on surface singularities and with Langois on toric singularities.

About the local topology of function-germs with a one-dimensional critical set

Hellen Monção de Carvalho Santana (University of São Paulo/Brazil)

Abstract: In this work, we consider two function-germs $f, g : (X, 0) \to (\mathbb{C}, 0)$ such that f has a stratified isolated singularity at the origin and g has a stratified one-dimensional critical set. We study the local topology of a deformation $\tilde{g}$ of g defined by $\tilde{g} = g + f^N$, where $N \gg 1$ and $N \in \mathbb{N}$ and we present the relation between the critical sets of g and $\tilde{g}$.

First steps on the simplicity of augmentations

Ignacio Breva Ribes (Universitat de València/Spain)

Abstract: The operation of augmentation can be found in almost all known classifications of simple map-germs. In general, the process of augmenting a simple singularity does not necessarily yield a simple augmentation. In this talk, we describe how to obtain the versal unfolding of an augmentation. This will allow us to characterise the simplicity of an augmentation in the case that the augmented map-germ has $\mathcal{A}_e$-codimension 1 or that the augmenting function is a Morse function. We also give a list of simple map-germs from $\mathbb{C}^4$ to $\mathbb{C}^4$. These, we conjecture, are all the simple augmentations that appear in these dimensions.

This is joint work with Raúl Oset Sinha (Universitat de València).

On the topological classification of circle-valued simple Morse-Bott functions

João Carlos Ferreira Costa (UNESP/Brazil)

Abstract: In this talk we give some recent results about the topological classification of circle-valued simple Morse-Bott functions defined on closed connected orientable surfaces. Here, topological classification means classification up to topological conjugacy. We give a complete topological invariant, so-called MB-Reeb graph, inspired on the generalized Reeb graph and on the topological type of the singular level sets of the functions. (A joint work with E.B. Batista and I.S. Meza-Sarmiento)

On Tjurina ideals

João Helder Olmedo Rodrigues (Brasil/Brazil)

Abstract: The Tjurina ideal of a germ of holomorphic function f on $\mathbb{C}^n$ is the ideal of germs generated by f itself and by its partial derivatives. It is denoted by $T(f)$. The ideal $T(f)$ gives the closed subscheme structure to the hypersurface singularity defined by f, being an object of central interest in singularity theory. In the talk we will present two simple, quite computable, operations on arbitrary ideals of germs of holomorphic functions that allow us to give necessary and sufficient conditions on an ideal of germs I, for the equation $I = T(f)$ to admit a solution f.

CMC surfaces in $\mathbb{R}^3$ with isolated singularities

José Edson Sampaio (Universidade Federal do Ceará/Brazil)

Abstract: In this talk, we will give a proof of the following version of Gulliver-Lawson's conjecture (Isolated Singularities Conjecture) for constant mean curvature (CMC) surfaces with isolated singularities: *Let M be a subset in $\mathbb{R}^3$ homeomorphic to a disc and such that $M \setminus \{0\}$ is a smooth CMC surface in $\mathbb{R}^3$. Then M is a smooth CMC surface.* With that proof, we obtain several consequences, for instance, we obtain the following: (1) A classification of a class of algebraic CMC surfaces in $\mathbb{R}^3$. We will show that a connected algebraic CMC surface in $\mathbb{R}^3$ with isolated singularities and a suitable condition of local connectedness is a plane or a finite union of round spheres and right circular cylinders touching at the singularities; (2) A CMC surface in $\mathbb{R}^3$ that is a topological manifold does not have isolated singularities; (3) A connected algebraic CMC surface in $\mathbb{R}^3$ with isolated singularities which is locally Lipschitz normally embedded must be a plane or a round sphere or a right circular cylinder.

The dual tree of a fold map germ from $\mathbb{R}^3$ to $\mathbb{R}^4$

Juan José Nuño Ballesteros (Universitat de València/Spain)

Abstract: Let $f\colon (\mathbb{R}^3, 0) \to (\mathbb{R}^4, 0)$ be an analytic map germ with isolated instability. Its link is a stable map $S^2 \to S^3$ which is obtained by taking the intersection of the image of f with a small enough sphere S^3_ϵ centered at the origin in $\mathbb{R}^4$. The simplest unstable singularities are those of fold type, in which $\dim_{\mathbb{R}} Q(f) = 2$, where $Q(f)$ is the Mather's contact algebra. When f is of fold type, we define a tree, that we call dual tree, that contains all the topological information of the link and we prove that in this case it is a complete topological invariant. As an application we give a procedure to obtain normal forms for any topological class of fold type.

On the geometry of curves in weighted projective planes and the Monodromy Conjecture for some surface singularities

Juan Viu Sos (Universidad Politecnica de Madrid/Spain)

Abstract: The Denef-Loeser topological zeta function is an analytic invariant of holomorphic map germs $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$, computed from embedded resolutions of f. The Monodromy Conjecture predicts that any pole of the zeta function is related with an eigenvalue of the monodromy of the Milnor fiber at some point of $f^{-1}(0)$.

In this talk, we introduce some recent techniques that we have developed for the study of these zeta functions for $\mathbb{Q}$-divisors over orbifold varieties and compositions of weighted blowing-ups. Finally, we give some applications of the previous techniques on the study of the Monodromy Conjecture for some surface singularities via the geometry of curves in embedded weighted projective planes.

This is a joint work Edwin LEON-CARDENAL (UNAM), Jorge MARTIN-MORALES (UNIZAR) and Wim VEYS (KU Leuven).

Relative Quasihomogeneity of Functions on Analytic Varieties

Konstantinos Kourliouros (Imperial College London/United Kingdom)

Abstract: In this talk we will investigate under which conditions the following relative analog of K. Saito's theorem holds true: equivalence of the relative Milnor and Tjurina numbers (also known as Bruce-Roberts numbers) of a function with respect to an analytic variety, is equivalent to the existence of a coordinate system such that both the function and the variety are quasihomogeneous with respect to the same system of positive rational weights.

This is a joint work with M. A. S. Ruas and C. B. Ausina.

On the Lipschitz geometry of quasi-homogeneous function-germs

Leonardo Câmara (UFES/Brazil)

Abstract: In this talk we recall the description of the moduli spaces of families of complex quasi-homogeneous function-germs on the plane under Lipschitz $\mathcal{R}$-equivalence and present some further developments for the real case. The first part of this talk is a joint work with M.A.S. Ruas ([CR]) and the second is part of a work in progress with A. Parusinski.

References

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Local invariants of divergence-free webs

Marcin Zubilewicz (Warsaw University of Technology/Poland)

Abstract: The aim of the talk is to highlight some features of the theory of regular webs in the geometry of volume-preserving transformations. The structures consisting of a smooth regular web and a nondegenerate volume form, called "divergence-free webs" by S. Tabachnikov in his paper on Lagrangian and Legendrian 2-webs, are more rigid than their ordinary differential-geometric counterparts. This rigidity manifests itself through the existence of local invariants which can be nontrivial even for classes of webs which are known to be always parallelizable. We will show how to construct two such invariants, both inspired by the holonomy and curvature of planar 3-webs defined by W. Blaschke and G. Thomsen in 1920s, and relate their triviality to the triviality of the underlying divergence-free web. We will also discuss their relationship to the curvature of Tabachnikov's connection, the coefficients of which occur inside the normal form of the volume element associated with a given divergence-free 2-web.

Joint work with W. Domitrz.

Singularities of Generic Line Congruences

Marcos Craizer (PUC-Rio/Brazil)

Abstract: Line congruences are 2-dimensional families of lines in 3-space. The singularities that appear in generic line congruences are folds, cusps and swallowtails. In this paper we give a geometric description of these singularities. The main tool used is the existence of an equiaffine pair defining a generic line congruence.

This is a joint work with Ronaldo A. Garcia.

Study of the curvature locus by the numerical range

Maria Garcia Monera (Universitat de València/Spain)

Abstract: For a m-submanifold immersed in $\mathbb{R}^{m+n}$, the geometry of the second fundamental form at a point is determined by the geometry of a set called the curvature locus. This concept is related to another concept, that of numerical range, of the field of linear algebra (or other

areas such as numerical calculus, probability, theoretical physics, etc ...). For low dimensions, the results can be translated from one area to another. In this talk we present the classification of the curvature locus by the numerical range of a 3-manifold immersed in $\mathbb{R}^5$.

The polar skeleton – vanishing polyhedra made explicit

Matthias Pablo Zach (TU Kaiserslautern/Germany)

Abstract: Lê's vanishing polyhedron for a complex analytic function $f\colon (X,0) \to (\mathbb{C},0)$ is an explicit realization of the vanishing topology – in theory! We discuss how the combinatorial information of the vanishing polyhedron can be extracted experimentally by a mixture of symbolic and numerical methods for suitable explicit examples in low dimensions.

Generic symmetry defect set of an algebraic curve

Michal Farnik (Jagiellonian University/Poland)

Abstract: Let $X \subset \mathbb{C}^{2n}$ be an n-dimensional algebraic variety. I will introduce the algebraic version of the generic symmetry defect set of X. Then I will compute its singularities for X_d being a generic curve of degree d in $\mathbb{C}^2$.

This is joint work with L.R.G. Dias and Z. Jelonek.

Axial curvatures for corank 1 singular n-manifolds in $\mathbb{R}^{n+k}$

Raúl Adrián Oset Sinha (Universitat de València/Spain)

Abstract: For singular n-manifolds in $\mathbb{R}^{n+k}$ with a corank 1 singular point at $p \in M^n_{sing}$ we define up to $l(n-1)$ different axial curvatures at p, where $l = \min\{n, k+1\}$. These curvatures are obtained using the curvature locus (the image by the second fundamental form of the unitary tangent vectors) and are therefore second order invariants. In fact, in the case $n = 2$ they generalise all second order curvatures which have been defined for frontal type surfaces. We relate these curvatures with the principal curvatures in certain normal directions of an associated regular $(n-1)$-manifold contained in M^n_{sing}. We obtain many interesting geometrical interpretations in the cases $n = 2, 3$. For instance, for frontal type 3-manifolds with 2-dimensional singular set, the Gaussian curvature of the singular set can be expressed in terms of the axial curvatures. Similarly for the curvature of the singular set when it is 1-dimensional. Finally, we show that all the umbilic

curvatures which have been defined for singular manifolds up to now can be seen as the absolute value of one of our axial curvatures.

This is joint work with J. Deolindo-Silva and P. Benendini Riul.

Newton lenses and algebraic complex singularities

Rémi Noël Langevin (Université de Bourgogne-Franche Comté/France)

Abstract: We renormalize, using suitable lenses, small domains of a singular holomorphic line field of $\mathbb{C}^2$ or plane field of $\mathbb{C}^3$ where the curvature of a plane-field is concentrated. At a proper scale the field is almost invariant by translations. When the field is integrable, the leaves are locally almost translates of a curve (in $\mathbb{C}^2$) or of a surface (in $\mathbb{C}^3$) that we will call *profile*. Singular surfaces in $\mathbb{C}^3$ will often demand multidirectional lenses. We also extend from dimension 2 to dimension 3 a result of Merle ([**?**]) concerning the contact order of generic polar curves with the singular level $\{f = 0\}$ when $\omega = df$. On the way we prove some classical results (Lê's carousels) on the knot $K = (\{f = 0\} \cap B_\epsilon(0, 0, 0))$ in dimension 2 and follow the track of graph manifolds in dimension 3.

Outer Lipschitz geometry of complex algebraic plane curves

Renato Oliveira Targino (Instituto Federal de Educação, Ciência e Tecnologia do Ceará/Brazil)

Abstract: One of the most natural questions in the investigation of a class of mathematical objects is the problem of classification of these objects. Here the classification problem is treated from the outer metric viewpoint: all subsets of the Euclidean space are considered equipped with the outer metric (i.e., the metric obtained by simply restricting the Euclidean metric). Our objects are complex algebraic plane curves and we obtain a complete classification of them up to a global bilipschitz homeomorphism for the outer metric.

Lipschitz (or metric) geometry of subanalytic singularities via horns.

Rodrigo Mendes Pereira (UNILAB/Brazil)

Abstract: Standard horns of given bases is a very natural object in order to produce metric distinctions along singular germs. On many works of Lipschitz geometry, it is an atom object, setting up as a base example for several invariants. In this talk, we systematize (probably know)

basic properties on horns, focusing on its arc orbits and its contacts. If times permits, we will bring some characterization of conical singularities.

It is a joint work with Dmitry Kerner and Lev Birbrair.

Focal set of a pure frontal

Samuel Paulino dos Santos (Unesp/Brazil)

Abstract: Let $f : (\mathbb{R}^2, 0) \to (\mathbb{R}^3, 0)$ be a C^∞ map germ. Then f is said to be a frontal if there exists a C^∞ map $\nu : (\mathbb{R}^2, 0) \to \mathbb{S}^2$ such that $\langle df_q(X), \nu(q)\rangle = 0$ holds for any $q \in (\mathbb{R}^2, 0)$ and $X \in T_q\mathbb{R}^2$. Moreover, a frontal f is called a *front* if the pair (f, ν) gives an immersion. We define *pure frontals* as surfaces being frontal, but not front in all its singular points. We study in this work the focal set of pure frontals. The focal set of a regular surface S is the loci of the centers of the spheres which have degenerated contact with S at each point. For a front f, Teramoto initiates the study of the focal set by observing that it can be treated as the singular loci of normal congruence of f. In this work, we use the same approach for pure frontals. However, besides the focal set, we get a new surface, called *normal ruled surface*. For the normal ruled surface, we give characterizations of singularities in terms of geometric invariants of the initial frontal defined on the set of singular points. For focal surfaces, we show the relation between certain singularities of them and geometric property of the given frontal. Moreover, we consider the behavior of Gaussian curvature of focal surfaces of the frontal with a 5/2-cuspidal edge. This is a joint work with Keisuke Teramoto.

Classification of Lipschitz A-simple Germs

Saurabh Trivedi (Indian Institute of Technology, Goa/India)

Abstract: We will present a complete classification of simple holomorphic function germs under Lipschitz A-equivalence.

Quadratic integrals of geodesic flow, webs, caustics and integrable billiards

Serguei Agafonov (São Paulo State University (UNESP)/Brazil)

Abstract: This talk links two seemingly unrelated topics: the geometry of planar webs and the dynamics of geodesic flow. We present a geometric interpretation of integrability of geodesic flow

by integrals, quadratic and cubic in momenta, in terms of the web theory. On surfaces, admitting quadratic integrals, we construct integrable billiards. For the flat case, these integrals are just conics (for quadratic integrals) and cubics (for cubic ones) in the dual space. Thus, the presented results are generalizations of some classical theorems, like Graf & Sauer, Poncelet theorems and of the properties of confocal conics to "integrable" metrics of non-constant curvature.

Symmetrization transforms of curves and surfaces

Stanislaw Tadeusz Janeczko (Warsaw University of Technology/Poland)

Abstract: Several point and space symmetrizations of curves and surfaces are introduced and one, the affinely invariant 'central symmetric transform' (CST) with respect to a given basepoint inside the convex plane curve, is investigated in detail. Examples for such curves include triangles, rounded triangles, ellipses, curves defined by support functions and piecewise smooth curves. Of particular interest is the region of basepoints for which the CST is convex (this region can be empty but its complement in the interior of the curve or surface is never empty). The (local) boundary of this region can have cusps and in principle it can be determined from a geometrical construction for the tangent direction to the CST. The new, measure of of symmetry of convex body based on an affine reflection phenomenon is introduced. The talk is based on common research with Peter Giblin.

Some classes of frontals and its representation formulas

Tito Alexandro Medina Tejeda (Universidade de São Paulo/Brazil)

Abstract: We characterize the extendibility of the normal curvature on frontals and we give a representation formula of this type of frontals. Also we give representation formulas for wavefronts on all types of singularities and others sub classes of these. Some applications to asymptotic curves and lines of curvature on frontals are made.

On singularities of the Gauss map components of surfaces in $\mathbb{R}^4$

Wojciech Domitrz (Warsaw University of Technology/Poland)

Abstract: This is a joint work with I. Hernández-Martínez and F. Sánchez-Bringas. The Gauss map of an immersion of an oriented surface into $\mathbb{R}^4$ takes values on the Grassmanian of oriented two-planes in $\mathbb{R}^4$. Since this manifold has a structure of a product of two spheres, this map splits

as a product of two components that take values on the sphere. We study the singularities of the components of the Gauss map and relate them to the geometric properties of the immersion. Moreover, we prove that the singularities are generically stable. Finally, we get some formulas of Gauss-Bonnet type involving the cusp points of the components.

5. Posters

Simple non-isolated singularities

Aline Bartel (Carl von Ossietzky Universität Oldenburg/Germany)

Abstract: It is a well known fact that non-isolated hypersurface singularities and complete intersection singularities cannot be simple. However, rigid ICMC2 singularities exist and are obviously simple. On my poster, I present results from work in progress toward a hierarchy of simple non-isolated ICMC2 singularities.

Hidden symmetries of plane curves and k-folding map-germs

Amanda Dias Falqueto (University of São Paulo/Brazil)

Abstract: We are going to show that all $\mathcal{A}$-simple singularities of germs of $\mathbb{C}, 0 \to C^2, 0$ can appear as singularities of k-folding map-germs (which is not the case for k-folding map-germs on surfaces, see [1]). [1] G. Peñafort Sanchis, F. Tari, *On k-folding map-germs and hidden symmetries of surfaces in the Euclidean 3-space.*

On the topology of the Milnor fibration and its application in topological robotics

Cesar Augusto Ipanaque Zapata (University of São Paulo/Brazil)

Abstract: In this paper we present some new recents results about the topology of the Milnor fibrations with a special attention to the topology of the fibers. In particular, we provide a short review on the existence of the Milnor fibrations in the real and complex cases that allows to compare our new results with the previous ones. Moreover, we exhibit an unexpected connection between the Milnor fibration theory, and current research trends in topological robotics. Joint work with Taciana Oliveira Souza.

Contact between a surface and a quadric

Fernanda Py Silva Cordeiro (PUC-Rio/Brazil)

Abstract: The goal of this work is to understand the contact between a surface and a quadric. Therefore, to build a continuous field of osculating quadrics to this surface. We use the quadric family that determines a Darboux direction. To promote a quadric that contains the best contact with the surface, we study the type of singularity of the contact function and verify that in an eliptic, hyperbolic or parabolic point, this quadric coincides with Moutard's quadric at the same Darboux direction. We aim to also understand the quadric field behavior near a quadratic point.

Singularities of 3-parameter Blaschke line congruences in $\mathbb{R}^4$

Igor Chagas Santos (USP/Brazil)

Abstract: In this work we define 3-parameter line congruences in $\mathbb{R}^4$, which is nothing but a 3-parameter family of lines over a hypersurface in $\mathbb{R}^4$. Locally, we write $\mathcal{C} = \{x(u), \xi(u)\}$, where $x : U \to \mathbb{R}^4$ (reference hypersurface) and $\xi : U \to \mathbb{R}^4 \setminus \{0\}$ (director hypersurface) are smooth, where $U \subset \mathbb{R}^3$ is open. We also investigate the singularities associated to a special class of congruences, called Blaschke affine normal congruences, which occurs when the reference hypersurface x is non-degenerate and the director hypersurface ξ is given by its Blaschke normal vector field. Our goal is to show that the generic singularities of

$$F_{(x,\xi)} : U \times I \to \mathbb{R}^4 \qquad (u,t) \mapsto x(u) + t\xi(u), \tag{1}$$

where I is an open interval, are the Lagrangian stable singularities, providing a positive answer to the conjecture presented in [1].

Joint work with: Débora Lopes (UFS) and Maria Aparecida Soares Ruas (ICMC).

References

[1] Izumiya, S., Saji, K., Takeuchi, N. Singularities of line congruences . Proc. of the Royal Society of Edinb., 133A, 1341-1359, 2003.

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Foliations on the plane: the quadratic-like case.

Ingrid Sofia Meza Sarmiento (UNIFEI/Brazil)

Abstract: In this poster we will consider functions from $\mathbb{R}^2$ to $\mathbb{R}$ given by $p(x,y) = r(x) + s(x)y + t(x)y^2$, where r, s, and t are real function of class C^1. These applications are called here by *quadratic-like functions*. Let p be a quadratic-like function. Then p will be a submersion if and only if the gradient of p, denoted by $\nabla p(x,y)$, in any point $(x,y) \in \mathbb{R}^2$ is not null. Note

that if $t(x) = 0$ for all $x \in \mathbb{R}$ then p is a linear-like submersion as studied in [1]. In this poster we will assume that t is not identically null. Given that p is a quadratic-like submersion then p defines a regular foliation in the plane given by the connected components of the level sets of p, denoted by $L_c = p^{-1}(c)$, $c \in p(\mathbb{R}^2)$. The main objective in this poster is to present a topological study of the regular foliations defined by quadratic-like submersions. We will show that for these kind of foliations two possible phenomena may occur at infinity: the vanishing and the splitting at infinity. Joint work with Francisco Braun (DM-UFSCar). References: [1] Meza-Sarmiento, I. S. and Braun, F., Topological equivalence of submersion functions and topological equivalence of their foliations on the plane: the linear-like case.

Preprint submitted for publication. https://arxiv.org/pdf/2203.01019.pdf

Surjectivity of linear operators and semialgebraic global diffeomorphisms

Jean Venato Santos (Universidade Federal de Uberlândia/Brazil)

Abstract: In this work, joint with F. Braun and L. R. G. Dias, we prove that a C^∞ semialgebraic local diffeomorphism of $\mathbb{R}^n$ with non-properness set having codimension greater than or equal to 2 is a global diffeomorphism if $n-1$ suitable linear partial differential operators are surjective. Then we state a new analytic conjecture for a polynomial local diffeomorphism of $\mathbb{R}^n$. Our conjecture implies a very known conjecture of Z. Jelonek. We further relate the surjectivity of these operators with the fibration concept and state a general global injectivity theorem for semialgebraic mappings which turns out to unify and generalize previous results of the literature.

Mixed maps and the Milnor-Hamm fibrations

Maico Felipe Silva Ribeiro (Universidade Federal do Espírito Santo/Brazil)

Abstract: We consider a new class of singularities called mixed maps from Oka?s class. In this new setting we prove the existence of Milnor?Hamm fibration on the tube and sphere. Moreover, we discuss the problem of existence of a Milnor vector field for this class. This is a join work with Fernando P. P. Reis (UFES) and Antonio A. do Espírito Santo (UFRB).

Newton polyhedra and Euler obstruction for isolated determinantal singularities

Maicom Douglas Varella Costa (Carl von Ossietzky Universität Oldenburg/Germany)

Abstract: Determinantal singularities form an important class of singularities, especially since they are considered a generalisation of complete intersections. There exist some very important invariants defined for singularities in general, such as the multiplicity, the Milnor number, the Euler obstruction, among others. These invariants often have geometric and/or topological meaning, while their computations are algebraic. In general, these computations are difficult to be done using their definitions, then finding formulas which allows us to compute them more easily is very significant. In this direction the Newton polyhedra plays an important role. Using elementary equivalence of matrices, we extend Esterov's formulas [1] to compute, using Newton polyhedra, the multiplicity of a determinantal singularity, the Euler characteristic of the Milnor fiber of a function $f : (X, 0) \to (\mathbb{C}, 0)$ defined on an Isolated Determinantal Singularity $(X, 0)$. As a consequence, we also obtain a formula for the local Euler obstruction of an isolated determinantal singularity in terms of Newton polyhedra.

References

[1] **A. Esterov**, *Determinantal singularities and Newton polyhedra*, Proceedings of the Steklov Institute of Mathematics, 2007, Vol. 259, pp. 16-34.

Generic sections of essentially isolated determinantal singularities

Nancy Carolina Chachapoyas Siesquén (UNIFEI/Brazil)

Abstract: We study the set of limits of tangent hyperplans to a determinantal variety $X^3 \subset \mathbb{C}^5$. We prove that a generic section of X is characterized by the fact that its Milnor number is minimum. The case of normal surface in $\mathbb{C}^N$ was studied by Jawad Snoussi in [**?**]. We also prove that if X is a d- dimensional EIDS and H and H' are strongly general hyperplans, the Milnor number of the surfaces $X \cap P$ and $X \cap P'$ are equal, if $P \subset H$ and $P' \subset H'$ are generic linear $d-2$ plans contained in H and H' . In the case that the generic section is a curve the result has been proved in [**?**].

From Milnor number to the Euler obstruction of a map on IDS

Raphael de Omena Marinho (Universidade de São Paulo/Brazil)

Abstract: In this work we study local invariants, as the Milnor number, polar multiplicity, the local Euler obstruction, and the Brasselet number, for the case of isolated determinantal singularities (IDS). We use the results of Grulha, Ruiz, and Santana [1] in order to relate the Euler obstruction of a map $f = (f_1, f_2) : (X, 0) \to (\mathbb{C}^2, 0)$ with isolated singularity, on an IDS $(X, 0)$ of dimension $d > 2$, to the vanishing Euler characteristic of a generic section $Y = X \cap f_2^{-1}(0)$. In the case of determinantal surfaces of codimension 2, the vanishing Euler obstruction coincides

with the Milnor number defined by Pereira and Ruas in [2], and we could compute explicitly the invariants; we also calculate for simple determinantal threefolds and fourfolds (codimension 2), considering the work of Zach [3].

References

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[3] M. Zach. Bouquet decomposition for determinantal Milnor fibers. Journal of singularities: 22, 190-204, 2020.

A discrete version of a four-vertex theorem for space curves

Samuel Pacitti Gentil (PUC-Rio/Brazil)

Abstract: We prove a discrete analogue of a certain four-vertex theorem for space curves. The smooth case goes back to the work of Beniamino Segre and states that a closed and smooth curve whose tangent indicatrix has no self-intersections admits at least four points at which its torsion vanishes. Our approach uses the notion of discrete tangent indicatrix of a (closed) polygon. Our theorem then states that a polygon with at least four vertices and whose discrete tangent indicatrix has no self-intersections admits at least four flattenings, i.e., triples of vertices such that the preceding and following vertices are on the same side of the plane spanned by this triple.

The realization problem of admissible graphs in coupled dynamics

Tiago de Albuquerque Amorim (USP/Brazil)

Abstract: In a coupled cell system, several individual dynamics (cells) are interacting in several ways. In [1] and [2] the authors formalize a general graph theory approach to investigate the dynamics of a coupled cell network, namely as a graph endowed with an input-equivalence relation on the set of vertices that enables a characterization of the admissible vector fields that rule the dynamics of the network. Recently we have noticed a way to go in the opposite direction: for $n \geq 2$, any mapping on $\mathbb{R}^n$ can be realized as an admissible vector field for some graph, where the number of vertices depending on n. This talk is devoted to present a method to construct all non-equivalent graphs, up to an appropriate equivalence relation, for which a mapping is an admissible vector field associated with. In applications, this means that, for a given set of individual systems to be coupled, one can predict how many and which couplings are possible, or not possible. A related and relevant problem to be investigated in the sequel is the relationship between invariant subspaces under this mapping and the synchronies of the associated networks. This is part of my on-going doctorate program at ICMC/USP.

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