Virtual Edition

16th

Celebrating 30 years of the workshop

International Workshop on Real & Complex Singularities

Book of Abstracts

ICMC, University of São Paulo, São Carlos, Brazil • November 23 to 30, 2020

Welcome!

This is a biennial conference realized by the **Singularity Theory Group**, São Carlos - Brazil. It has been established as one of the key events on singularity theory, algebraic geometry, bifurcation theory and related areas. This edition celebrates 30 years of the conference and it will be entirely online.
It brings together internationally renowned and young researchers to report on their recent achievements, to exchange ideas and to address trends of research in a highly stimulating environment.
If you encounter any difficulties during your stay, please feel free to contact any of us from the Organizing Committee.

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We are here to help you with anything!

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Contents

Program

The program of the Workshop consists of:

- **Opening session:** short talks from some of the many fellow researchers who have supported the workshop throughout its trajectory.
- **19 live plenary talks** of 50 minutes by invited speakers. These talks will be broadcast live by YouTube and Facebook.
- **35 parallel recorded talks** of 25 minutes, whose links will be available at all times at the workshop media (webpage and YouTube). Questions and comments must be made via e-mail message or by the comments place below the video on YouTube.
- **17 posters** (recorded short videos or written files), available at all times at the workshop media (webpage, YouTube and Facebook).
- **Gathering session:**

All times in the next table are Brasilia-BR time zone (GMT-3).

MONDAY 23th

9:00am - 10:00am	**Opening Session**
10:10am - 11:10am	**Carolina Araujo** (IMPA, Brazil), Symmetries in algebraic geometry and Cremona transformations
11:10am - 11:40am	*WebCoffee*
11:40am - 12:40pm	**Terry Gaffney** (Northeastern University, USA), Equisingularity and non-isolated singularities
2:00pm - 2:30pm	**Claus Hertling** (University of Mannheim, Germany)
2:45pm - 3:25pm	**Xiping Zhang** (SCMS, Fudan University, China), Local Euler Obstructions of Reflective Group Orbits **Stavros Anastassiou** (University of Patras, Greece), Local classification of dynamical systems on the Liouville plane
3:25pm - 4:00pm	*Coffee break*
4:00pm - 4:30pm	**Eder Leandro Sanchez Quiceno** (ICMC-USP, Brazil), Real algebraic links in the 3-sphere associated with mixed polynomials **Rustam Sadykov** (Kansas State University, USA), Minimizing Crit(M)

TUESDAY 24th

9:00am - 10:00am	**Raúl Oset-Sinha** (Universitat de València, Spain), The axial curvature for corank 1 singular surfaces
10:10am - 11:10am	**Alexandre A. P. Rodrigues** (Universidade do Porto, Portugal), Unfolding a Bykov attractor: from an attracting torus to strange attractors
11:10am - 11:40am	*WebCoffee*
11:40am - 12:40pm	**Bruna Oréfice Okamoto** (Universidade Federal de São Carlos, Brazil), Isolated determinantal singularities and function germs over them
2:00pm - 2:30pm	**Guillermo Peñafort Sanchis** (Universidad Nacional Autónoma de México, Mexico), k-folding mappings and robust features of surfaces in $\mathbb{R}^3$
2:45pm - 3:25pm	**Roberto Giménez Conejero** (Universitat de València, Spain) Graham Reeve (Liverpool Hope University, United Kingdom)
3:25pm - 4:00pm	*Coffee break*
4:00pm - 4:30pm	**Shunsuke Ichiki** (Tokyo Institute of Technology, Japan), A refinement of transversality theorems **Yanlin Li** (Hangzhou Normal University, China), Tangent developables and Darboux developables of framed curves

WEDNESDAY 25th

9:00am - 10:00am	**Ana Bravo** (Universidad Autónoma de Madrid, Spain), On the intrinsic nature of resolution invariants
10:10am - 11:10am	**Jörg Schürmann** (University of Münster, Germany), A non-characteristic pullback formula for motivic Chern and Hirzebruch classes
11:10am - 11:40am	*WebCoffee*
11:40am - 12:40pm	**Luis Renato Gonçalves Dias** (Universidade Federal de Uberlândia, Brazil), Bifurcation values of algebraic mappings and Jacobian conjecture
2:00pm - 2:30pm	**Vyacheslav Sedykh** (Moscow Center for Continuous Mathematical Education, Russia), Swallowtail, Whitney umbrella and convex hulls
2:45pm - 3:25pm	**João Carlos Ferreira Costa** (IBILCE-UNESP, Brazil), The cone structure theorem for map germs **Reinaldo Roberto Rosa** (INPE, Brazil), Singularities in cosmological manifolds: some mathematical and computational challenges
3:25pm - 4:00pm	*Coffee break*
4:00pm - 4:30pm	**León Kushner** (UNAM, Mexico), Quadratics, Cubics and Quartics **Konstantinos Kourliouros** (ICMC-USP, Brazil), Singularities of functions and multi vector fields of maximal degree

THURSDAY 26th

9:00am - 10:00am	**José Edson Sampaio** (Universidade Federal do Ceará, Brazil), A version of the Mumford's Theorem on regularity of normal complex surfaces in high dimension
10:10am - 11:10am	**Daniel Duarte** (Universidad Autónoma de Zacatecas, Mexico), Nash blowup and higher Nash blow ups in positive characteristic
11:10am - 11:40am	*WebCoffee*
11:40am - 12:40pm	**Tamás László** (Babes, Bolyai University, Cluj-Napoca, Romania), On the delta invariant of curves on rational surface singularities
2:00pm - 2:30pm	**Ingrid Sofia Meza Sarmiento** (UFSCar, Brazil), Swallowtail, Whitney umbrella and convex hulls **Thiago da Silva** (UFES, Brazil), The double structure on arbitrary rings, modules and scheme
2:45pm - 3:25pm	**Filipe Bellio da Nóbrega** (École Normale Supèrieure de Lyon, France), Osculating conics of a smooth curve on the projective plane **Maria Michalska** (UFC, Brazil), Constructive approach to optimal degree inequalities on semialgebraic sets
3:25pm - 4:00pm	*Coffee break*
4:00pm - 4:30pm	**Taciana Oliveira Souza** (UFU, Brazil), Milnor fibrations and its topology **Piotr Mormul** (University of Warsaw, Poland), Singularities of Special Multi flaps at the crossroads of Algebraic Geometry and Differential Geometry

FRIDAY 27th

9:00am - 10:00am	**Meral Tosun** (Galatasaray University, Turkey), Free divisors coming from triple quivers
10:10am - 11:10am	**Kenta Hayano** (Keio University, Japan), Stability of non-proper functions
11:10am - 11:40am	*WebCoffee*
11:40am - 12:40pm	**José Seade** (Universidad Nacional Autónoma de Mexico, Mexico), On the boundary of the Milnor fiber for non-isolated singularities
2:00pm - 2:30pm	**Rafaela Soares de Carvalho** (UFSCar, Brazil), μ-constant deformations of function on ICIS **Keisuke Teramoto** (Institute of Magthematics for Industry, Kyushu University, Japan), Behavior of principal curvatures of frontals near non front singular points
2:45pm - 3:25pm	**Nhan Nguyen** (Basque Center for Applied Mathematics, Spain), Regular projection in o-minimal structures **Dominik Wrazidlo** (Institute of Mathematics for Industry, Kyushu University, Japan), On the rational homotopy type of intersection spaces
3:25pm - 4:00pm	*Coffee break*
4:00pm - 4:30pm	**Leonardo Câmara** (UFES, Brazil), Some aspects of complex function germs on the plane **Fernando P. P. Reis** (UFES, Brazil), Milnor-Hamm fibration for mixed maps

SATURDAY 28th

9:00am - 10:00am	**Wojciech Domitrz** (Warsaw University of Technology, Poland), Singular improper affine spheres from a given Lagrangian submanifold
10:10am - 11:10am	**Laurentiu Maxim** (University of Wisconsin-Madison, USA), Homological duality: jumping loci, propagation, realization
11:10am - 11:40am	*WebCoffee*
11:40am - 12:40pm	*Gathering Session*
2:00pm - 2:30pm	**Fernando Antoneli** (UNIFESP, Brazil), Homeostasis, Singularities and input-output networks **Maicom Varella** (UFSCar, Brazil), Newton Polyhedra and Whitney Equisingularity for IDS
2:45pm - 3:25pm	**Miguel A. de la Rosa Castillo** (CONACYT-UJAT, Mexico), K. Saito-Hertling pairing as a bridge between Grothendieck and Poincaré dualities **Douglas Hilário da Cruz** (UFG, Brazil), Knots and asymptotic lines

MONDAY 30th

9:00am - 10:00am	**André Belotto da Silva** (Aix-Marseille Université, France), A proof of Gabrielov's rank Theorem
10:10am - 11:10am	**Marcelo Escudeiro Hernandes** (Universidade Estadual de Maringá, Brazil), The analytic classification of plane curves
11:10am - 11:40am	*WebCoffee*
11:40am - 12:40pm	**Jean-Paul Brasselet** (Aix-Marseille Université, France), With Euler, a 50 years journey per minute, from Athens to São Carlos
2:00pm - 2:30pm	**Juan Antonio Moya Pérez** (Universitat de València, Spain), Finite C0-determinacy of real analytic map germs with isolated instability **Samuel dos Santos** (IBILCE-UNESP, Brazil), Geometry of bifurcation sets of unfoldings of functions with singularity A3, A4 and D4
2:45pm - 3:25pm	**Bárbara Karolline de Lima Pereira** (UFSCar, Brazil), The Bruce Roberts Number of a Function on an Isolated Hypersurface Singularity **Tito Medina Tejeda** (ICMC-USP, Brazil), Extendibility and boundedness of invariants on singularities of wavefronts
3:25pm - 4:00pm	*Coffee break*
4:00pm - 4:30pm	**Dmitry Kerner** (Ben Gurion University, Israel), $C^r/C^{r,\mathrm{Lip}}$-trivialization of equi-resolvable families of isolated singularities

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Murilo André de Jesus Zigart - Universidade Estadual de Campinas, Brazil

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16th

Celebrating 30 years of the workshop

International Workshop on Real & Complex Singularities

Plenary Talks

ICMC, University of São Paulo, São Carlos, Brazil • November 23 to 30, 2020

Symmetries in Algebraic Geometry and Cremona Transformations

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Abstract: When studying a projective variety X, one usually wants to understand its symmetries. The structure of the group of automorphisms of X encodes relevant geometric properties of X. In birational geometry, however, the notion of automorphism is too rigid, and it is more natural to consider birational self-maps. Birational self-maps of the projective space $\mathbb{P}^n$ are called Cremona transformations. Describing the structure of the group of Cremona transformations of the plane is a challenging problem that goes back to the 19th century. In higher dimensions, much less is known. I will discuss a joint project with Alessio Corti and Alex Massarenti, where we explore the birational geometry of Calabi-Yau pairs, and use them to construct special subgroups of the 3-dimensional Cremona group.

On the intrinsic nature of resolution invariants

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Abstract: An algorithmic resolution of singularities of a variety X defined over a field k, consists on giving a procedure to select the centers to blow up,

$$X = X_0 \leftarrow X_1 \leftarrow \ldots \leftarrow X_n,$$

so that X_n is non-singular.
When the characteristic of k is zero, or the dimension of X is low and k is perfect, an algorithmic resolution can be found by defining *resolution invariants*. These are numerical functions that help classifying the singularities of X according to their complexity, a classification that ultimately leads to the construction of the centers in the previous sequence of blow ups. One of the the most important resolution invariants is *Hironka's order function*. In this talk we will look beyond the resolution process and study Hironaka's order function and other resolution invariants from a different perspective, as will be explained in the talk. Among other results, this will lead us to find connections between the resolution invariants and the space of arcs of X. The contents of the talk are based on a series of papers with different coauthors: C. Abad, A. Benito, S. Encinas, B. Pascual-Escudero and O.E. Villamayor.

A proof of Gabrielov's rank Theorem

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Abstract: This talk concerns Gabrielov's Rank Theorem, a fundamental result in local complex and real analytic geometry, proved in the 1970's. Contrasting with the algebraic case, it is not in general true that the analytic rank of an analytic map (that is, the dimension of the analytic Zariski closure of its image) is equal to the generic rank of the map (that is, the generic dimension of its image). This phenomenon is behind several pathological examples in local real analytic geometry. Gabrielov's Rank Theorem provides a formal condition for the equality to hold.
In spite of its importance, the original proof is considered very difficult. There is no alternative in the literature, besides work by Tougeron, which is itself considered very difficult. I will present our recent work in collaboration with Octave Curmi and Guillaume Rond, where we provide a proof of Gabrielov's Rank Theorem. Indeed, we develop formal geometric techniques, inspired by ideas of Gabrielov's and Tougeron's, in order to clarify the difficult part of the proof.

With Euler, a 50 years journey per minute, from Athens to São Carlos

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Abstract: In this lecture we take a trip back in time (from 480 BC to 2020 AD) and space (from Athens to São Carlos, via Stockholm, Berlin, Paris, São José do Rio Preto).
We will ask ourselves:

- Did Plato in Athens and Archimedes in Syracuse know Euler's formula for polyedra?
- Did Descartes discover the formula before Euler?
- Did Cauchy proved the formula ?
- Which characteristic did Poincaré do?
- Why does Euler appear in singularity theory?
- Why do researchers in São Carlos want to uncover the secrets of Euler's obstruction?

Bifurcation values of algebraic mappings and Jacobian Conjecture

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Abstract: We present results related to the bifurcation set of algebraic mappings. Also, we present topological approaches to the Jacobian Conjecture and we present a theorem for the non-properness set of a non-singular polynomial mapping $f : \mathbb{C}^n \to \mathbb{C}^n$. In particular, our result extends a bidimensional result of Lê and Weber to higher dimensions. The talk is based on the collaboration with F. Braun (UFSCar) and J. Venato-Santos (UFU).

Singular improper affine spheres from a given Lagrangian submanifold

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Abstract: This is the joint work with Marcos Craizer and Pedro de M. Rios. Given a Lagrangian submanifold L of the affine symplectic $2n$-space, one can canonically and uniquely define a center-chord and a special improper affine sphere of dimension $2n$, both of whose sets of singularities contain L. Although these improper affine spheres (IAS) always present other singularities away from L, they may also present singularities other than L which are arbitrarily close to L, the so called singularities "on shell". These on-shell singularities possess a hidden $\mathbb{Z}_2$ symmetry that is absent from the off-shell singularities. We study these canonical IAS obtained from L and their on-shell singularities, in arbitrary even dimensions, and classify all stable Lagrangian/Legendrian singularities on shell that may occur for these IAS when L is a curve or a Lagrangian surface.

Nash blowup and higher Nash blowups in positive characteristic

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Abstract: In this talk we present some results concerning Nash blowups over fields of positive characteristic. We show that a normal variety is non-singular if and only if its Nash blowup is an isomorphism, extending a theorem by A. Nobile. We also extend a result by R. Toh-Yama which shows that higher Nash blowups do not give a one-step resolution of the A_3-singularity. This is a joint work with Luis Núñez Betancourt.

Equisingularity and Non-isolated Singularities

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Abstract: In this talk we will look at some of the issues involved in the hunt for invariants of non-isolated singularities which ensure the Whitney equisingularity of a family of non-isolated singularities.

Stability of non-proper functions

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Abstract: In this talk, we will give a sufficient condition for (strong) stability of non-proper smooth functions (with respect to the Whitney C^∞–topology). We introduce the notion of end-triviality of smooth mappings, which concerns behavior of mappings around the ends of the source manifolds, and show that a Morse function is stable if it is end-trivial at any point in its discriminant. We further show that a Morse function $f : N \to \mathbb{R}$ is strongly stable (i.e. there exists a continuous mapping $g \mapsto (\Phi_g, \phi_g) \in \mathrm{Diff}(N) \times \mathrm{Diff}(\mathbb{R})$ such that $\phi_g \circ g \circ \Phi_g = f$ for any g close to f) if (and only if) f is quasi-proper. As an application of this result, we will give an explicit example of a strongly stable but not infinitesimally stable function.

The Analytic Classification of Plane Curves

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Abstract: In this talk, we present a solution to the problem of the analytic classification of germs of plane curves with several irreducible components. Our algebraic approach follows percursive ideas of Oscar Zariski and as a subproduct allow us to recover some particular cases found in the literature. This is a joint work with M. E. Rodrigues Hernandes.

Homological duality: jumping loci, propagation, realization

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Abstract: I will discuss recent progress on the study of homological duality properties of complex algebraic manifolds, with a view towards the projective Singer-Hopf conjecture. (Joint work with Y. Liu and B. Wang.)

On the delta invariant of curves on rational surface singularities

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Abstract: In this talk we discuss our recent result which shows that the delta invariant of a complex curve germ is the periodic constant of the Campillo, Delgado and Gusein-Zade's Poincaré series associated with the curve. This can be used to prove simple formulae for the delta invariant in the case when the curve is embedded in a rational surface singularity. Joint work with J.I. Cogolludo-Agustín, J. Martín-Morales and A. Némethi.

Isolated determinantal singularities and function germs over them

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Abstract: Let $(X, 0)$ be an isolated determinantal singularity and $f : (X, 0) \to \mathbb{C}$ a holomorphic function germ with isolated singularity. We define the vanishing Euler characteristics and the top polar multiplicities of $(X, 0)$ and of $(f\hat{}-1(0), 0)$ and the Milnor number of f. We relate these invariants to the local Euler obstruction of f. Furthermore, we study how these invariants are related to equisingularities of families of isolated determinantal singularities and of function germs on isolated determinantal singularities.
Joint work with D. A. H. Ament, R. S. de Carvalho, J. J. Nuño-Ballesteros and J. N. Tomazella.

The axial curvature for corank 1 singular surfaces

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Abstract: We will define the axial curvature at a point p in a surface in $\mathbb{R}^3$ where p is a corank 1 singularity of the parametrisation. We will study properties of this curvature and relate it to other known curvatures. In fact, we will show that this curvature generalizes the singular curvature defined for frontal type singularities. We will investigate geometrical applications of this curvature and give some future prespectives.
Joint work with Kentaro Saji.

Unfolding a Bykov attractor: from an attracting torus to strange attractors

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Abstract: We present a mechanism for the emergence of strange attractors in a two-parametric family of differential equations acting on a three-dimensional sphere. When both parameters are zero, its flow exhibits an attracting heteroclinic network (Bykov network) made by two 1-dimensional and one 2-dimensional separatrices between two hyperbolic saddles-foci with different Morse indices. After slightly increasing both parameters, while keeping the one-dimensional connections unaltered, we focus our attention in the case where the two-dimensional invariant manifolds of the equilibria do not intersect.

We show the existence of many complicated dynamical objects, ranging from an attracting quasi-periodic torus, Newhouse sinks to Hénon-like strange attractors, as a consequence of the Torus Bifurcation Theory (developed by Afraimovich and Shilnikov). Under generic and checkable hypotheses, we conclude that any analytic unfolding of a Hopf-zero singularity (within an appropriate class) contains strange attractors. We also discuss the case of the existence of rank-one strange attractors (developed by Q. Wang and L.-S.Young) for this model.

A version of the Mumford's Theorem on regularity of normal complex surfaces in high dimension

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Abstract: It is well known that if a closed (i.e., compact and without boundary) 3-manifold M is simply connected then M is diffeomorphic to $\mathbb{S}^3$, before its proof this result was known as Poincaré's conjecture. In 1961, before its proof, D. Mumford proved that a normal complex analytic surface X with simply connected link at 0 need to be smooth at 0 and, in particular, its link at 0 is diffeomorphic to $\mathbb{S}^3$. In this talk, we prove a version of Mumford's result in high dimension. More precisely, if $X \subset \mathbb{C}^n$ is a Lipschitz normally embedded (LNE) complex analytic set and $d = \dim X$, we prove that the following statements are equivalent:

(1) X is a homology $2d$-manifold (i.e., $H_*(X; X \setminus \{x\}) = H_*(\mathbb{R}^{2d}; \mathbb{R}^{2d} \setminus \{0\})$ for all $x \in X$), locally linearly contractible (LLC) and its link at 0 is simply connected;

(2) X has no choking cycles at 0 and its link at 0 is $(2d-2)$-connected;

(3) The Hausdorff limit $X_0 := \lim_{t \to 0^+} (\frac{1}{t} X) \cap \mathbb{S}^{2n-1}$ is $(2d-2)$-connected;

(4) The tangent cone of X at 0 is a linear subspace of $\mathbb{C}^n$.

(5) X is smooth at 0.

As a consequence, we obtain that if Y is the link at 0 of a d-dimensional LNE complex analytic set X (not necessary with isolated singularity), X is LLC and a homology $2d$-manifold and Y is $(2d-2)$-connected then Y is diffeormophic to $\mathbb{S}^{2d-1}$. Another consequence is the following: Let $X \subset \mathbb{C}^n$ be a d-dimensional LNE complex analytic set with an isolated singularity at $0 \in X$ and assume that its link at 0 is $(2d-2)$-connected. If there exists $K_0 \in [0, +\infty)$ such that the sectional curvature of $X_t := (\frac{1}{t} X) \cap \mathbb{S}^{2n-1}$, denoted by K_{X_t}, satisfies $0 \leq |K_{X_t}| \leq K_0$, for all small enough $t > 0$, then X is smooth at 0. In particular, we obtain that the link of X at 0 is diffeomorphic to $\mathbb{S}^{2d-1}$.
This is a joint work with Alexandre Fernandes.

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A non-characteristic pullback formula for motivic Chern and Hirzebruch classes

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Abstract: We give an introduction and overview about the different functorial theories of characteristic classes of singular spaces in the complex algebraic context. Then we explain a new non-characteristic pullback formula for motivic Chern and Hirzebruch classes. Applications include an intersection formula for transversal intersections, as well as a proof of a weak form of a conjecture about the agreement of the intersection homology Hirzebruch class (for the parameter $y = 1$) with the homology L-class for a complex projective algebraic variety.

On the boundary of the Milnor fiber for non-isolated singularities

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Abstract: Let $f : (\mathbb{C}^{n+1}, p) \to (\mathbb{C}, 0)$ be a holomorphic function germ with critical point at p, set $V = f^{-1}(0)$, let $L_f = V \cap \mathbb{S}_\varepsilon$ be its link, and recall that L_f determines fully the topology of V. The Milnor fibers of f can be regarded as the family of local non-critical levels $F_t := f^{-1}(t) \cap \mathbb{B}_\varepsilon$ with $t \neq 0$, which degenerate to the special fiber $F_0 := V \cap \mathbb{B}_\varepsilon$ as t approaches 0. There is a vast literature studying how this degeneration $F_t \rightsquigarrow F_0$ takes place. Simultaneously, as the F_t degenerate to F_0, their boundaries ∂F_t "converge" to the link $L_f = \partial F_0$. If p is an isolated critical point of f, all the ∂F_t are ambient isotopic to L_f. Yet, if p is a non-isolated critical point, then the ∂F_t with $t \neq 0$, are a family of real analytic manifolds converging to the link L_f, which now is singular. In this talk we study the degeneration $\partial F_t \rightsquigarrow L_f$. This is joint work with Aurelio Menegon and Marcelo Aguilar, and it springs from previous work by Randell, Siersma, Michel-Pichon-Weber, Némethi-Szilard and Fernández de Bobadilla-Menegon.

Free divisors coming from triple quivers

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Abstract: We will introduce triple quivers and we construct a new root system for the triple quivers. And, we show that, for each root, we obtain a linear free divisor. This is a joint work with A.Altintas Sharland and K.Nakamoto.

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Local classification of dynamical systems on the Liouville plane

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Abstract: We study vector fields of the plane preserving the form of Liouville. We present their local models up to the natural equivalence relation, and describe local bifurcations of low codimension. To achieve that, a classification of univariate functions is given, according to a relation stricter than contact equivalence. We discuss, in addition, their relation with strictly contact vector fields in dimension three. Analogous results for diffeomorphisms are also given.

Homeostasis, Singularities and Input-Output Networks

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Abstract: Homeostasis occurs in a biological or biochemical system when some output variable remains approximately constant as some input parameters vary over some range. Recently, Golubitsky and Stewart [Homeostasis, Singularities and Networks. J. Math. Biol. 74 (2017) 387-407] introduced the notion of 'infinitesimal homeostasis' allowing the use of implicit differentiation and singularity theory to study homeostasis in systems of differential equations. Networks consisting of nodes and unidirectional arrows encode systems of differential equations. Nodes correspond to state variables and arrows indicate which nodes are coupled to which. What distinguishes a network system of differential equations from a large system of differential equations is the capability to keep track of the output from each node individually. Hence, infinitesimal homeostasis is related to occurrence of 'singularities' at individual nodes. In this talk we explain a new approach to the study of the combinatorial structure and classification of homeostasis in 'input-output networks', that is, networks where we keep track of the output from a fixed node, as well as the node(s) that depend on the external input parameters.
Joint work with Martin Golubitsky, Ian Stewart, Zhengyuan Huang, Yangyang Wang and João Luiz de Oliveira Madeira.

Some aspects of complex function-germs on the plane

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Abstract: In this talk we address the study of the moduli spaces of the $\mathcal{R}$-equivalence of families of complex analytic function-germs on the complex plane. We consider the Lipschitz and analytic standpoints.

μ-constant deformations of functions on an ICIS

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Abstract: Greuel, in 1986, studied properties of the constancy of the Milnor number of a deformation $f_t\colon (\mathbb{C}^n,0) \to (\mathbb{C},0)$ of a holomorphic function germ with isolated singularity resulting in the following theorem:
Theorem: Let $f\colon (\mathbb{C}^n,0) \to (\mathbb{C},0)$ be a holomorphic function germ with isolated singularity at the origin. For any deformation $F\colon (\mathbb{C}\times\mathbb{C}^n,0) \to (\mathbb{C},0)$ of f the following statements are equivalent
(1) F is μ-constant;
(2) For every holomorphic curve $\gamma\colon (\mathbb{C},0) \to (\mathbb{C}\times\mathbb{C}^n,0)$

$$\nu\left(\frac{\partial F}{\partial t}\circ\gamma\right) > \inf\{\nu\left(\frac{\partial F}{\partial x_i}\circ\gamma\right) \mid i=1,\ldots,n\},$$

(where ν denotes the usual valuation of a complex curve);
(3) Same statement as in (2) with " $>$ replaced by " $\geq$;

(4) $\dfrac{\partial F}{\partial t} \in \overline{J}$, where $\overline{J}$ denotes the integral closure of the Jacobian ideal $J = \left\langle \dfrac{\partial F}{\partial x_1},\ldots,\dfrac{\partial F}{\partial x_n}\right\rangle$ as an ideal in $\mathcal{O}_{n+1}$;

(5) $\dfrac{\partial F}{\partial t} \in \sqrt{J}$, where $\sqrt{J}$ denotes the radical of J;

(6) $v(J) = \mathbb{C}\times\{0\}$ near $(0,0)$.

In this work, we studied this result for families $f_t\colon (X,0) \to (\mathbb{C},0)$, where $(X,0)$ is an ICIS.

Joint work with Bruna Oréfice Okamoto and João Nivaldo Tomazella. Departamento de Matemática, Universidade Federal de São Carlos - UFSCar.

The cone structure theorem for map germs

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Abstract: In this talk we investigate some aspects of the topological classification of map germs. It is known that the topological structure of a map germ $f : (\mathbb{R}^n, 0) \to (\mathbb{R}^p, 0)$ can be determined by the topological type of its associated link. When $f^{-1}(0) = \{0\}$ and f satisfies a generic condition (which includes the case that f is finitely determined) follows from Fukuda's cone structure theorem that f is C^0-$\mathcal{A}$-equivalent to the cone of its link. In this work we show the cone structure theorem for finitely determined map germs but now when $f^{-1}(0) \neq \{0\}$. To do this we introduce the notions of link diagram and generalized cone. It follows that the link diagram is well defined up to topological equivalence and that f is topologically equivalent to the generalized cone of its link diagram. This is a joint work with E.B. Batista and J.J. Nuño-Ballesteros.

K. Saito-Hertling pairing as a bridge between Grothendieck and Poincaré dualities

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Abstract: For an isolated hypersurface singularity $f : (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ we will show a relationship between bilinear forms defined on the Milnor algebra coming from Grothendiek residue and bilinear forms in the canonical Milnor fiber of vanishing cohomology which come from the polarization ("Poincaré duality") of Hertling-Steenbrink. These bilinear forms take into acount two nilpotent maps: the first, consider the map given by multiplication by f, and the second, use the nilpotent part, N, of the monodromy in cohomology. The principal objective of the talk consists in explaining how the K. Saito-Hertling pairing allows us to state such relations.

Monodromy of germs of analytic functions without fixed points

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Abstract: In this joint work with J.J. Nuño-Ballesteros and Lê Dũng Tráng we prove that, given $f : (X, x) \to (\mathbb{C}, 0)$ such that $f \in \mathfrak{m}^2 O_{X,x}$, there is a geometric local monodromy of f without fixed points and we give an application of this fact in a broad context.

A geometric monodromy appears every time we have a local trivial fibration over S^1, say $f : U \to S^1$. Broadly speaking, it is a map of a fiber $F = f^{-1}(x_0)$ onto itself that is defined by taking F to give a *loop around* S^1.This is the situation of $f : (X, x) \to (\mathbb{C}, 0)$ such that $f \in \mathfrak{m}^2 O_{X,x}$ and the fibration induced by taking a small enough circumference around 0, in this case is called local geometric monodromy. Finally, we use it to prove that, in a broad context, the critical points of a family of functions from a family of complex analytic sets cannot split along the family.

This generalizes two theorems of the second coauthor, one stated for $\mathbb{C}^n$ instead of X and other for hypersurfaces; and gives an alternative proof of a result of A'Campo, that the Lefschetz number is zero.

The integral monodromy of isolated quasihomogeneous singularities

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Abstract: In 1972 Peter Orlik proposed a conjecture which says roughly that the isomorphism class of the pair (Milnor lattice, integral monodromy) of an isolated quasihomogeneous hypersurface singularity takes the simplest possible shape. This year (2020) in September we proved the conjecture for all iterated Thom-Sebastiani sums of chain type singularities and cycle type singularities (invertible polynomials). This surpasses all known results. We have a relative result (the Thom-Sebastiani sum of any two singularities which satisfy the conjecture, satisfies the conjecture) and results for the chain type and cycle type singularities. In the background, there are several algebraic results.

Knots and asymptotic lines

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Abstract: In the first part of this talk, I will present and discuss the following theorem: the trefoil knot is a hyperbolic asymptotic line of a suitable plane field. Pictures will be used to illustrate the concepts and results. In the second part of the talk, I will present a generalization of the theorem. Joint work with Ronaldo Garcia.

A refinement of transversality theorems

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Abstract: In this talk, I introduce a transversality theorem from the viewpoint of Hausdorff measures. The transversality theorem is an improvement of the basic transversality result and its strengthening which was given by Mather.

$C^r/C^{r,\mathbf{Lip}}$-trivialization of equi-resolvable families of isolated singularities

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Abstract: Take a (complex-analytic) hypersurface germ with an isolated singularity, and consider its deformation that is topologically trivial. Sometimes one can ensure a Lipschitz (or even C^k) trivialization. These trivializations are traditionally constructed via controled vector fields, or using the metric geometry.
For equi-resolvable families we construct a much better behaved trivialization: it is identity outside of a small ("hornic") neighbourhood of the hypersurface, real analytic in the interior of this neighbourhood, flat on the boundary of this neighbourhood, and of C^k, $C^{k,Lip}$-type at the origin. Then we give tight lower bounds on this k. Even in the classical cases of weighted-homogeneous/Newton-non-degenerate singularities this improves the known results.

Singularities of Functions and Multi-Vector Fields of Maximal Degree

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Abstract: We present local classification results for isolated singularities of functions with respect to a multi-vector field (Nambu structure) of maximal degree, in a neighbourhood of a smooth point of its degeneracy hypersurface. The results depend on a logarithmic version of the Brieskorn-Sebastiani theorem, which guarantees the finiteness and freeness of the corresponding deformation module. This relates the functional moduli of the classification problem with the integrals of logarithmic forms along the vanishing cycles of the complement of the Milnor fibers of the restriction of the function on the degeneracy hypersurface of the Nambu structure, inside the Milnor fibers of the function itself.

Quadratics, cubics and quartics

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Abstract: In this talk we study the action of special subgroups of the the group of invertible matrices on the quadratics in two, three and four variables, cubics in two and three variables and quartics in two and three variables and their stabilizers. The Lie subgroups are Lie groups of dimension 3, and 6 respectively.

Tangent developables and Darboux developables of framed curves

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Abstract: In this presentation, I will talk about the singularity properties of special developable surfaces of framed curves in Euclidean 3-space. Based on several works of S. Izumiya, G. Ishikawa and M. Takahashi, we will give the relationship between types of singularities of tangent developable surfaces of framed curves and frame curvature functions. That enables us to recognize the type of singularities of tangent developable surfaces of framed curves from the frame curvature functions of the frame curves. This is a joint work with Professor Zhigang Wang.

The Bruce Roberts Number of a Function on an Isolated Hypersurface Singularity

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Abstract: Let $(X,0)$ be an isolated hypersurface singularity defined by $\phi\colon (\mathbb{C}^n,0)\to(\mathbb{C},0)$ and $f\colon(\mathbb{C}^n,0)\to\mathbb{C}$ such that the Bruce-Roberts number $\mu_{BR}(f,X)$ is finite. In this work we prove that $\mu_{BR}(f,X)=\mu(f)+\mu(\phi,f)+\mu(X,0)-\tau(X,0)$, where μ and τ are the Milnor and Tjurina numbers respectively of a function or an isolated complete intersection singularity. We also prove that the logarithmic characteristic variety $LC(X,0)$ is Cohen-Macaulay, both results generalize [1]. This is a joint work with J. J. Nuño-Ballesteros (Universitat de Valencia, SPAIN), B. Oréfice-Okamoto, (UFSCar, BRAZIL) and J.N. Tomazella, (UFSCar, BRAZIL).

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Extendibility and boundedness of invariants on singularities of wavefronts

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Abstract: The aim of this work is to provide necessary and sufficient conditions for the extendibility and boundedness of Gaussian curvature, Mean curvature and principal curvatures near all types of singularities on fronts. Some known results on non-degenerated singularities are generalized. Also we study the divergence of these geometrical invariants and we show how this is tightly related to a property of approximation of fronts by parallel surfaces.

On the relation between topological equivalence of submersion functions and their foliations in the real plane

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Abstract: Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a smooth function such that $\partial f/\partial x$ and $\partial f/\partial y$ have no common zeros in $\mathbb{R}^2$, i.e., f is a smooth *submersion*. We denote by $\mathscr{F}(f)$ the *regular foliation* of $\mathbb{R}^2$ whose leaves are the connected components of f.
As usual we say that two functions $f, g : \mathbb{R}^2 \to \mathbb{R}$ are *topologically equivalent* (resp. o-*topologically equivalent*) if there exist homeomorphisms (resp. orientation preserving homeomorphisms) $h : \mathbb{R}^2 \to \mathbb{R}^2$ and $\ell : \mathbb{R} \to \mathbb{R}$ such that

$$f \circ h = \ell \circ g,$$

Clearly the topological equivalence of f and g guarantees the topological equivalence of $\mathscr{F}(f)$ and $\mathscr{F}(g)$, but the converse is not true, in general, even in the polynomial case. By considering a class of submersions wide enough in order to contain the polynomial submersions of degree less than or equal to 3 (this includes Brower example), we provide necessary and sufficient conditions in order that the converse implication is also valid.

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Constructive approach to optimal degree on semialgebraic sets

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Abstract: We show some constructive methods to determine the optimal real exponent d in an inequality $|f| \leq C|g|^{d/\deg g}$ on a semialgebraic set S, where f and g are polynomials, and C is some positive constant. For a fixed polynomial f, the optimal exponent that satisfies this inequality for some g and for points of S outside a compact is called the relative degree of f on S. Similarly as in the case of Łojasiewicz exponent, this optimal d is a rational number. We present an algorithmic method to find the grading introduced by relative degree on the ring of polynomials, when the set S is defined by quasihomogeneous polynomials. We apply this result to all subsets of the real plane and give an algorithmic characterization of the grading. In the lecture we indicate several other applications.

Singularities of special multi-flags at the crossroads of Algebraic Geometry and Differential Geometry

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Abstract: There exist different approaches to the singularities of special multi-flags, also (misguidely) called 'generalized Goursat flags'. They live in Monster Towers, also called Semple Towers.
Colley et al have concluded, in Michigan Math. J. 166 (2017), a series of earlier algebro-geometric constructions, by several authors, of fine stratifications of the stages of Semple Towers, eventually producing so-called 'RV-classes of singularities'.
In the meantime the present author constructed in the stages of Monster Towers, in SIGMA 5 (2009), the so-called 'singularity classes', using purely differential and Lie-algebraic tools.
It had been generally believed that the former classes (much more numerous) were a refinement of the latter ones. This belief now turns out to be false. The two approaches appear, to a sizeable degree, to mutually complement each other.
And - important - complementary appear the very languages used in the two approaches. In fact, the charts used on the DG side are [by tradition] called 'Extended Kumpera-Ruiz'. Virtually the same charts used in the AG side are called 'C-charts'.
However, the motivations underlying charts' constructions in AG and DG are so distant that there has been, until recently, no identification procedure in sight.
Such a procedure has been found over the last summer. With an effective translation going now both ways between the EKS's and C-charts, the singularities seen on the AG side can now be interpreted on the DG side, and vice versa.

Finite $\mathcal{C}^0$-determinacy of real analytic map germs with isolated instability

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Abstract: Let $f\colon (\mathbb{R}^n,0)\to(\mathbb{R}^p,0)$ be a real analytic map germ with isolated instability. We prove that if $n=2$ and $p=2,3$ f is finitely C^0-determined. This result can be seen as a weaker real counterpart of Mather-Gaffney finite determinancy criterion.

Regular projection in o-minimal structures

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Abstract: We prove that for a given definable set $X \subset \mathbb{R}^n$ with empty interior there always exists a definable bi-Lipschitz homeomorphism $h : \mathbb{R}^n \to \mathbb{R}^n$ such that $h(X)$ has a finite set of regular projections (in the sense of Mostowski). A consequence of this result is the existence of definable regular covers for definable sets.

Osculating conics of a smooth curve on the projective plane

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Abstract: The Tait-Kneser theorem is a well known result of differential geometry which states that the osculating circles of a plane curve with monotonic curvature and no inflection points are disjoint and nested. Therefore, an arc with no vertex gives rise to an interesting foliation of the region of the plane delimited by its largest and smallest osculating circle. In this talk, we will investigate the analogous result for osculating conics. It is already known that the osculating conics of a curve with no sextactic or inflection points are also disjoint and nested. However, we will show that the relative position of two such conics is actually more restricted than that, they are in some sense "convexly nested".

k-folding mappings and robust features of surfaces in $\mathbb{R}^3$

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Abstract: Given a smooth surface M in 3-space (real or complex) and an integer $k \geq 2$, we can associate a holomorphic map-germ $(\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$ of the form $(x, y) \mapsto (x, y^k, f(x, y))$ to any point $p \in M$ and any plane π through p. These map-germs encode the order k local symmetries of M, and are called k-folding mappings because their image is the result of performing a k-th order fold on (the complexification of) M along the plane π. We give a complete topological classification of the k-folding mappings exhibited by generic manifolds for $k \geq 3$, and relate the presence of different classes to the extrinsic differential geometry of the surface M. The case of $k = 2$ was studied by Bruce and Wilkinson, who showed that changes in the type of folding mappings exhibited capture the subparabolic and ridge curves, as well as the more degenerate S_3, B_3 and C_3-points on them. By letting k to be arbitrary, we show that local symmetry captures, in a unified approach, all known robust features related to contact with planes, spheres and lines, and it gives some new robust features.

Contact of surfaces with circles and Euclidean invariants

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Abstract: I discuss some recent (and not so recent) results on the contact between circles and smooth surfaces in three space. This is joint work with Peter Giblin and Ricardo Uribe-Vargas.

Milnor-Hamm fibration for Mixed maps

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Abstract: We consider a new class of singularities called *mixed maps* from Oka's class. In this new setting we prove the existence of Milnor-Hamm fibration on the tube and sphere. Moreover, we discuss the problem of existence of a Milnor vector field for this class.

Singularities in cosmological manifolds: mathematical and computational challenges

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Abstract: One of the major problems in contemporary cosmological models is the premise that starts from Friedmann's singularity. In this talk we describe what are the main mathematical and computational problems related to the concept of cosmological singularity and how some of them can be solved by topological and geometric properties associated with the gravitational field described by the theory of general relativity.
The mathematical concept of cusp cosmology is presented including its main advantages and limitations which can be solved by purely topological principles.

Minimizing Crit(M)

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Abstract: In a joint work with Stanislav Trunov we prove that the minimal number of critical points of functions on a closed connected manifold M is the same as the minimal number of balls in a ball filling of M. As an application, we use trisections of 4-manifolds to study the minimal number of critical points of functions on 4-manifolds.

Real algebraic links in the 3-sphere associated with mixed polynomials

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Abstract: Let $f : \mathbb{C}^2 \to \mathbb{C}$ be a mixed polynomial, i.e., a complex polynomial of variables $(u, \bar{u}, v, \bar{v})$ such that $f(0) = 0$ and that f has an isolated singularity at the origin. Then, associated with f, we have the well-defined link, which is the intersection of the mixed hypersurface $f^{-1}(0)$ with a sphere of small radius. Such a link is called a real algebraic link. Classification and characterization of real algebraic links are still open. In this talk, a new construction of families of real algebraic links is discussed and how properties of mixed polynomials are used to generate such families is explained.
This work is a part of my PhD's thesis under the supervision of professor Raimundo Nonato Aráujo dos Santos, it was devolved during a visit to the Institute of Mathematics for Industry, Kyushu University under the supervision of my co-advisor Osamu Saeki, the research was supported by grant 2019/11415-3, São Paulo Research Foundation (FAPESP) and was partially financed by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brazil (CAPES) – Finance Code 001.

Geometry of bifurcation sets of unfoldings of functions with singularity A_3, A_4, and $D_4^{\pm}$

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Abstract: We study the geometry of the bifurcation set of unfoldings of some specific functions with corank 1 and 2. More precisely, we suppose the bifurcation set is either a cuspidal edge, a swallowtail, a pyramid or a purse, i.e, we suppose that the functions have singularities of type A_2, A_3, D_4^- or D_4^+ at the origin, respectively. In the two first cases, we calculate geometric invariants of the singularity at the origin which depends on the coefficients of the initial unfolding. In the case of $D_4^{\pm}$ singularities, we study the number of parabolic, ridge, and subparabolic curves through the singularity of the bifurcation set. We also study the behavior of parabolic curves on the bifurcation set, which depends on the coefficients of the initial unfolding.

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Swallowtail, Whitney umbrella and convex hulls

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Abstract: One of singularities of the convex hull of a generic hypersurface in $\mathbb{R}^4$ leads to a generic sewing of two famous surfaces, the swallowtail and the Whitney umbrella, along their self-intersection lines. Germs of all such sewings at the common end-point of the self-intersection lines are diffeomorphic to each other with respect to diffeomorphisms of the ambient space.

The double structure on arbitrary rings, modules and schemes

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Abstract: In previous work, Gaffney and da Silva dealt with bi-Lipschitz equisingularity of a family of analytic varieties from an algebraic point of view, using notions as the integral closure of modules and the double.
In this talk I present and define the double in a purely algebraic way, for modules over arbitrary rings, motivated mainly by the fibered product of (affine) schemes.

Milnor fibrations and its topology

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Abstract: We present some new recent results about the topology of the Milnor fibrations with a special attention to the topology of the fibers. In particular, we provide a short review on the existence of the Milnor fibrations in the real and complex cases that allows to compare our new results with the previous ones.

Behavior of principal curvatures of frontals near non-front singular points

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Abstract: Recently, there are several studies on curves and surfaces with singular points from differential geometric viewpoints. In particular, geometric properties of frontals and wave fronts have been actively researched. In this talk, we deal with frontals with non-front singular points such as a cuspidal cross cap or a 5/2-cuspidal edge. We show relationship between behavior of principal curvatures of frontals near non-front singular points and geometric invariants defined at such singular points. Moreover, umbilicities of non-front singular points and behavior of principal vectors will be given. This is a joint work with Kentaro Saji (Kobe University).

Newton Polyhedra and Whitney Equisingularity for Isolated Determinantal Singularities

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Abstract: Using Newton polyhedra, we present a condition that guarantee the Whitney equisingularity of a family of Isolated Determinantal Singularities $\{(X_t, 0)\}$. Moreover, given X an Isolated Determinantal Singularity, we use topological and geometric information presented by Pereira and Ruas [2] and Nuño-Ballesteros, Oréfice-Okamoto and Tomazella [1], to establish a Lê–Greuel type formula for germs $f : (X, 0) \to (\mathbb{C}, 0)$ and $g : (X, 0) \to (\mathbb{C}, 0)$, with stratified isolated singularity.

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On the rational homotopy type of intersection spaces

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Abstract: Banagl's method of intersection spaces allows to modify certain singular spaces near the singular set in such a way that the rational Betti numbers of the modified spaces satisfy generalized Poincaré duality in analogy with Goresky-MacPherson's intersection homology theory. In this talk, we present a new perspective on the topic by using tools of rational homotopy theory. Our approach yields uniqueness of commutative models for intersection spaces up to weak equivalence, and a new geometric interpretation of generalized Poincaré duality as an intersection pairing.

Local Euler Obstruction of Recursive Group Orbits

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Abstract: The local Euler obstructions of a projective variety and the Euler characteristics of its linear section with given hyperplanes are key geometric invariants in the study of singularity theory. Despite their importance, in general is very hard to compute them. In this paper we consider a special type of singularity: the recursive group orbits. They are the group orbits of a sequence of G_n representations V_n satisfy certain assumptions.
We introduce a new invariant called the c_{sm} invariant, and use it to give formula to the local Euler obstructions and sectional Euler characteristics of such orbits. In particular, the matrix rank loci are examples of recursive group orbits. Thus as application, we explicitly compute these geometry invariants for ordinary, skew-symmetric and symmetric rank loci. Our method is algebraic, thus works for algebraically closed field of characteristic 0.

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Tjurina number and semi-roots of plane branches

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Abstract: The topological classification of plane curves can be translated in terms of discrete invariants. Analytical classification is a finer problem and cannot be solved using a discrete set of invariants. However, we can fix a topological class and stratify it using analytical invariants. Here we present results that relate the analytical invariants of a semi-root to the respective analytical invariants of the curve.

$\mathscr{A}_e$-Codimension and Image Milnor Number

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Abstract: Let $f : (\mathbb{C}, S) \to (\mathbb{C}^2, 0)$ be a map germ of finite $\mathscr{A}_e$-codimension. D. Mond proves in (3) a relation between the image Milnor number and the $\mathscr{A}_e$-codimension,

$$\mathscr{A}_e\text{-codim}(f) \leq \mu_I(f),$$

with equality if and only if f is weighted homogeneous.
Inspired by the previous inequality, the authors consider in (1) a finite map germ $f : (X, 0) \to (\mathbb{C}^2, 0)$ of degree 1 onto its image $(Y, 0)$, where $(X, 0)$ is a plane curve. They can consider the $\mathscr{A}_e$-codimension, $\mathscr{A}_e$-codim(X, f), in the sense of Mond and Montaldi (4) because $(X, 0)$ is an isolated complete intersection singularity (ICIS). They define the image Milnor number in this case and obtain the similar relation,

$$\mathscr{A}_e\text{-codim}(X, f) \leq \mu_I(f),$$

with equality if and only if $(Y, 0)$ is weighted homogeneous.
In (2), the authors consider map germs $f : (X, 0) \to (\mathbb{C}^2, 0)$, where $(X, 0)$ is an ICIS of dimension one. They generalize the concept of image Milnor number to this case. Moreover, if $(X, 0)$ is irreducible and both $(X, 0)$ and f are weighted homogeneous (with the same weights), they show that

$$\mathscr{A}_e\text{-codim}(X, f) = \mu_I(f).$$

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Apparent contour of surfaces in $\mathbb{R}^3$ and extensions of Koenderink's formula

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Abstract: The orthogonal projection $P_{\mathbf{v}}$ of a regular surface $M \subset \mathbb{R}^3$ along a direction $\mathbf{v}$, is locally a germ of application of the plane in the plane that measures the contact of M with lines and is singular at $p \in M$ if and only if the direction of projection is tangent to M at p. The set of points $p \in M$ where $P_{\mathbf{v}}$ is singular is a curve called *contour generator* of M along the direction $\mathbf{v}$ and the image of the orthogonal projection of this set is also a curve called the *profile* or *apparent contour* along the direction $\mathbf{v}$. Geometric information of M can be obtained from the singularities of $P_{\mathbf{v}}$. In particular, the Koenderink's theorem relates the Gaussian curvature of M at the point p of the contour generator with the curvature of the apparent contour at $P_{\mathbf{v}}(p)$ and the normal curvature along the direction of projection (taking regular points of the apparent contour).

Realization of MB-Reeb graphs associated to circle-valued Morse-Bott functionss

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Abstract: In this work we investigate the problem of the realization of a given graph as the MB-Reeb graph $\mathcal{R}(f)$ associated to a circle-valued Morse-Bott function $f : \mathbb{M}^2 \to S^1$ with finitely many critical points, where $\mathbb{M}^2$ is a smooth oriented manifold.

Nash Transformation of an EIDS

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Abstract: Essentially isolated determinantal singularities (EIDS) is a generalization of isolated complete intersection singularities (ICIS). In this work we study the Nash transformation of EIDS and give sufficient conditions for this transformation to be smooth.

Curvature lines of the double torus

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Abstract: We describe de ν-lines of curvature of an embedding of a double torus into $\mathbb{R}^4$, defined as the link of the real part of the Milnor fibration of a polynomial, where ν is the gradient. Throughout this analysis, we present a complete description of the foliation of lines of curvature of the embedding, defined as the image of the stereographic projection of this link into $\mathbb{R}^3$.

Versality of central projections of regular surfaces

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Abstract: A center of a central projection is often regarded as a viewpont and central projections naturally form unfoldings with viewpoints as unfolding parameters. We present criteria of singularity types of central projections of regular surfaces in $\mathbb{R}^3$ whose $\mathcal{A}_e$-codimension is 3 or less. We also present criteria of "versality" of this central projection unfoldings.

Singularities of parallels to tangential maps of frontals

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Abstract: A frontal in a Euclidean space is called *normally flat* if the normal connection of the normal bundle is flat. Given a normally flat frontal in a Euclidean space, we define its parallel frontals. Then, in particular, we show that the tangent surface to a frontal curve generated by tangent lines to the curve is normally flat, provided to it is a frontal, and show that the parallels to the tangent surface turn to be right equivalent to the tangent surfaces of parallels to the original curve. Moreover we give the exact classification of generic singularities appearing in parallels to tangent surfaces of generic curves in 3 and 4 dimensional Euclidean spaces.

Understanding the worlds of higher dimensional closed and smooth manifolds of several classes via explicit fold maps

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Abstract: This poster concerns studies on understanding several classes of higher dimensional closed and smooth manifolds via construction of explicit fold maps, by the author. Fold maps are higher dimensional versions of Morse functions. As related existing studies, studies on the class of special generic maps, which contains Morse functions on closed manifolds with exactly two singular points, characterizing spheres topologically except 4-dimensional cases, and canonical projections of unit spheres as simplest examples, are known and interesting, for example. It has been shown that special generic maps restrict the topologies and the differentiable structures of the manifolds strictly in various scenes, by Saeki, Sakuma, Wrazidlo etc.. In this poster, we present explicit construction of fold maps of new suitable classes and algebraic topological and differential topological properties of the manifolds explicitly. Especially, we construct explicit fold maps on higher dimensional manifolds of suitable classes such as the class of 7-dimensional closed and simply-connected manifolds. This class has been attractive since the discovery of Milnor's exotic spheres and is still actively studied by Crowley, Kreck, and so on, via concrete bordism theory, for example. The aim of this presentation is to present studies of a new type: understanding such classes in constructive ways via explicit fold maps.

Newton non-degenerate μ-constant deformations admit simultaneous embedded resolutions

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Abstract: Let $\mathbb{C}_o^{n+1}$ denote the germ of $\mathbb{C}^{n+1}$ at the origin. Let V be a hypersurface germ in $\mathbb{C}_o^{n+1}$ and W a deformation of V over $\mathbb{C}_o^m$. Under the hypothesis that W is a Newton non-degenerate deformation, in this work, we prove that W is a μ-constant deformation if and only if W admits a simultaneous embedded resolution. This result gives a lot of information about W, for example, the topological triviality of the family W and the fact that the natural morphism $(W(\mathbb{C}_o)_m)_{red} \to \mathbb{C}_o$ is flat, where $W(\mathbb{C}_o)_m$ is the relative space of m-jets. On the way to the proof of our main result, we give a complete answer to a question of Arnold on the monotonicity of Newton numbers in the case of convenient Newton polyhedra.

Horrocks-Mumford holomorphic distributions

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Abstract: This work is devoted to the study of codimension two holomorphic distributions on projective space P4, whose tangent or conormal sheaves are Horrocks-Mumford. Our first goal is to describe the geometry of the singular scheme of these distributions. We prove that the singular scheme is a smooth, reduced, irreducible (hence connected) curve and we calculate its Rao module. We show that such distributions are non-integrable. Finally, we describe the Moduli space of these distributions, proving that such space is an irreducible quasi-projective variety and we calculate its dimension.

The geometry of a family of quadratic systems with two multiple singularities

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Abstract: Consider the family $\mathbf{QsnSN_{11}}$ of real planar quadratic polynomial differential systems which have a finite saddle–node, a simple finite singularity and an infinite saddle–node formed by the coalescence of a finite and an infinite singularities. This family, modulo the action of the affine group and time rescaling, depends on four real parameters, and the bifurcation diagram of its closure with respect to a specific normal form is the three–dimensional real projective space (see [1]). In this poster we intend to present a summary of the study of this bifurcation diagram.
Joint work with Professors Joan Carles Artés (UAB), Regilene Delazari dos Santos Oliveira (ICMC–USP), and Alex Carlucci Rezende (DM–UFSCar).

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Information Geometry from Singularity Theory Viewpoint

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Abstract: The dually flat structure of Amari-Nagaoka is particularly highlighted in Information Geometry, a successfully developing field in the interaction of differential geometry, statistical science and information theory. In this poster presentation, we report part of our recent project, which proposes a novel generalization of the dually flat structure for singular models from the viewpoint of Lagrange and Legendre singularity theory. This work is motivated by various interests from Frobenius manifolds to Deep Learning.

New classes of mixed functions without Thom regularity

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Abstract: A function $f : \mathbb{C}^n \to \mathbb{C}$ is called *mixed polynomial function* if $f(z) = f(\mathbf{z}, \bar{\mathbf{z}}) = \Sigma_{\nu,\mu} c_{\nu,\mu} \mathbf{z}^\nu \bar{\mathbf{z}}^\mu$, where $c_{\nu,\mu} \neq 0$, $\mathbf{z}^\nu := z_1^{\nu_1} \cdots z_n^{\nu_n}$ and $\bar{\mathbf{z}}^\mu := \bar{z}_1^{\mu_1} \cdots \bar{z}_n^{\mu_n}$ for n-uples $\nu = (\nu_1, ..., \nu_n)$, $\mu = (\mu_1, ..., \mu_n) \in \mathbb{N}^n$.
In this work, we use this class in order to consider the problem of existence of local fibration structures verifying the main regularity conditions considered in recent literature, such as the well known *Thom regularity condition* and the *ρ-regularity*.
We extend some results previously known for the class of singularities $f\bar{g}$, with f and g being holomorphic functions, for a product fg with f and g mixed functions. We use these results in order to prove when the sum of convenient mixed functions have Thom regularity.
We adapted a technique used in the Section 5.1 of [2] in order to give a criterion to decide when a mixed function is not Thom regular at $V_f := f^{-1}(0)$.
Next we consider a special class of mixed function called *mixed simple ł-map*, for short *MSL function*, which was introduced in [1]. We slightly improved the Proposition 5.5 of [1] by creating a method to construct new examples of MSL functions from the product of two mixed functions.
Finally, we construct infinite examples of maps with isolated critical value, with local Milnor fibrations on the tube and sphere and without Thom regularity.

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Projective structures on surfaces of finite type

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Abstract: We will define singular projective structures of the Fuchsian type on surfaces and prove the existence theorem of singular projective structures of the Fuchsian type with given monodromy representation $\rho : \pi_1(S^*) \to PSL_2(\mathbb{C})$, where S^* is a Riemann surface of finite type, and we will study the relation between isomonodromic projective structures and the flip of a fiber. From an analytical point of view, we will calculate the Schwarzian derivative of Fuchsian projective charts and solutions of Schwarzian equations given a meromorphic quadratic differential with double pole. We will make a local study of the geometry/topology of the developing map that defines the projective structure around a cusp and we will extend surgery of movement of branch points when one of the singularities is Fuchsian type. Finally, we will give algebra-geometric interpretations for these projective structures where we will obtain formulas involving the surface's topological invariants and analytical invariants of foliation and fiber bundle.

3-Parameter Line Congruences in $\mathbb{R}^4$

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Abstract: A 3-parameter line congruence in $\mathbb{R}^4$ is a 3-parameter family of lines over a hypersurface in $\mathbb{R}^4$. An important class of line congruences is the class of normal congruences, i.e. a 3-parameter line congruence such that there is a regular hypersurface whose normal vectors are parallel to the lines of the congruence. The family of normal lines over a hypersurface is a classical example of a normal congruence, called an exact normal congruence. Here, we use methods of Singularity Theory to show that the generic singularities of 3-parameter line congruences are different from the generic singularities of 3-parameter normal congruences. Furthermore, we have that the generic singularities of normal congruences are the same that the generic singularities of exact normal congruences.

Normal forms for Hamiltonian systems with symmetries

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Abstract: The aim is to present an algebraic method to determine normal forms for Hamiltonian systems in the presence of symmetries. Such a method takes into account the Hamiltonian condition and symmetry types of the original system from the beginning of the process in order to preserve them in the normal form. The procedure is an adaptation of the methods developed by Elphick et al. [2] and by Baptistelli, Manoel and Zeli [1] combined with the techniques of normal forms of Hamiltonian systems. The results presented here are contained in [3].

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