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On Zariski equisingularity

Adam Parusinski

Nice - France

We give a report on recent result on Zariski equisingularity including a construction of an arc-wise analytic stratification and the proof of Whitney's fibering conjecture. We also discuss the relation between Zariski equisingularity and Lipschitz stratification as well as several open problems.

Stratification of the Dynamics of Three-Dimensional Real Vector Fields

C. Alonso-González

University of Alicante - Spain

It is well known that a neighborhood of a singular point of an analytic vector field in the plane having a characteristic trajectory splits into a finite union of hyperbolic, parabolic and elliptic sectors. In this talk we present an analogous result for analytic three dimensional real vector fields under non degeneracy conditions. This is a joint work with Fernando Sanz Sánchez (University of Valladolid)

Image Milnor number and A_e -codimension for maps between curves

DAIANE ALICE HENRIQUE AMENT

Universidade Federal de São Carlos - Brazil

We define the image Milnor number of $f: (X,0) \to (\mathbb{C}^2,0)$, where (X,0) is a space curve ICIS and f is a finite map germ of degree 1 onto its image (Y,0). If $(X,0) \subset (\mathbb{C}^n,0)$ is an irreducible weighted homogeneous singularity curve and $f: (X,0) \to (\mathbb{C}^2,0)$ is a map germ finite and weighted homogeneous with the same weights of (X,0). We show that $\mathcal{A}_{e^-}(X,f) = \mu_I(f)$, where $\mathcal{A}_{e^-}(X,f)$ is the \mathcal{A}_{e^-} codimension, i.e., the minimum number of parameters in a versal deformation and $\mu_I(f)$ is the image Milnor number, i.e., the number of vanishing cycles in the image of a stabilisation of f. This is a joint work with J. J. Nuño Ballesteros and J. N. Tomazella.

Which surface singularities are Lipschitz normally embedded?

ANNE PICHON

Aix Marseille University - France

We will present two large classes of surface singularities which are Lipschitz normally embedded. The first one is the class of minimal surface singularities: any minimal surface

singularity is Lipschitz normally embedded, and they are the only rational surface singularities with this property. I will explain how this result is closely related with the resolution of singularities. This result is a joint work with Walter Neumann and Helge Pedersen. The second class of examples is among superisolated singularities. This result is a joint work with Filip Misev.

Abstract: This talk is related with Helge Pedersen's talk and should be scheduled after his talk if both are selected.

An algebraic characterization of the Boy surface

FRANÇOIS APÉRY

UHA - France

In 1984, I obtained a parametrization of the Boy surface by eliminating the Whitney umbrellas of the Steiner surface using the so-called hyperbolic confluence of pairs of singularities. As a result, the Boy surface appeared to be a real algebraic surface of degree six. However, the construction was a mix of geometry, differential topology and singularity theory. In this talk I want to investigate the question in the complex algebraic geometry field. We intend to characterize the Boy surface as an complex algebraic surface subjected to natural conditions.

Configurations of singularities of quadratic systems: A tree with 1879 branches

JOAN C. ARTÉS, JAUME LLIBRE, DANA SCHLOMIUK AND NICOLA VULPE

Autonoma Barcelona - Spain

Along the last ten years, D. Schlomiuk, N. Vulpe, J. Llibre and J.C. Artés have been working in the geometrical classification of singularities (finite and infinite) of planar quadratic differential systems. Seven partial papers have been produced and finally a book containing all them plus lattest most generic cases has been completed. We present these results altogether with a Mathematica tool that allows an easy way to study families of quadratics systems with some parameters and determine easily the bifurcation diagram related to the singularities.

Hopf bifurcation at infinity and dissipative vector fields of the plane

BEGOÑA ALARCÓN COTILLAS

Universidade Federal Fluminense - Brazil

We present one-parameter families of differentiable (not necessarily C^1) planar vector fields for which the infinity reverses its stability as the parameter goes through zero. These vector fields are defined on the complement of some compact ball centered at the origin and have isolated singularities. They may be considered as linear perturbations at infinity of a vector field with some spectral property, for instance, dissipativity. We also address the case concerning linear perturbations of planar systems with a global period annulus. It is worth noting that the adopted approach is not restricted to consider vector fields which are strongly dominated by the linear part. Moreover, the Poincaré compactification is not applied in this work. Actually, we strongly focus on the change of the sign of an index defined at infinity. Joint work with Roland Rabanal (Pontificia Universidad Católica del Perú).

This paper (DOI 10.1090/proc/13462) pretends to extend to the differentiable case a previous work published in DCDS-A by the first author (joint with C. Gutierrez and V. Guíñes). The authors of DCDS-A guarantee the change of stability at infinity of a one-parameter family of C^1 -vector fields as the parameter goes through zero. This behaviour was presented as Hopf bifurcation at infinity.

The L-Class of Singular Spaces: Old and New Results

MARKUS BANAGL Heidelberg University - Germany

Hirzebruchs L-class is well-known to occupy a central role in the topological classification of high-dimensional manifolds. The L-class of singular spaces can be assigned a similar role, but much less is known about its behavior, and only a relatively small number of concrete calculations have been carried out. We review the most generally known context in which L-classes are definable and provide an overview of the current level of understanding. The focus will ultimately lie on some recent results obtained through an interaction of Ranicki's L-homology, singular bordism and complex Hodge theory.

On affine curvature lines of surfaces

MARTIN BARAJAS S.

Universidad Distrital FJC - Colombia

In this work we study the affine curvature lines of surfaces near to isolated singularities of the associated field of directions (affine umbilic points), and the case when the affine curvature line is closed (affine cycles).

This is a joint work with Ronaldo Garcia and Marcos Craizer.

A chinese

JEAN-PAUL BRASSELET

CNRS - France

That will be a "Brasil-China" lecture : I will explain in an elementary way the contribution of Wu Wen Tsun to characteristic classes of singular varieties and influence on brasilian mathematics.

Generic geometry of stable maps of 3- manifolds into \mathbb{R}^4

C. CASONATTO, M. C. ROMERO FUSTER AND R. WIK ATIQUE

UFU - Brazil

Singularity Theory techniques have proven to be useful in the description of the geometrical properties of submanifolds in Euclidean space. This is due to the fact that the geometrical invariants of a submanifold at a point can be given in terms of the geometrical models (k-spheres and k-planes) that better approach the submanifold at the considered point and their contacts can be meassured through the singularity type of conveniently defined functions and maps on the submanifold. In particular, the families of height and distance squared functions on a submanifold M of \mathbb{R}^n respectively contain all the information on the contacts of M with the hyperplanes and hyperspheres of \mathbb{R}^n and the description of their corresponding singularities leads to an alternative characterization of classically known geometric properties, as well as to new relevant concepts and results. Basic illustrative references are the pioner papers due to I. Porteous [2] and C. T. C. Wall [3]. An interesting aspect of this viewpoint is the possibility of applying it not only to smooth submanifolds defined by immersions, but also to a more general class of geometrical objects, such as the images of stable maps. The case of a stable map from the plane to 3-space was treated by J. W. Bruce and J. M. West [1], who analyzed the behaviour of a crosscap with respect to its contacts with planes in \mathbb{R}^3 .

In this work we study the generic contacts of the 3D-crosscap (image of a germ in the \mathcal{A} -orbit of (x, y, z^2, yz)) with hyperplanes in \mathbb{R}^4 as a consequence of the classification of submersion germs $(\mathbb{R}^4, 0) \to (\mathbb{R}, 0)$ under the $_V\mathcal{K}$ -equivalence relation, where V is the standard 3D-crosscap, that is, the image of $(x, y, z) \mapsto (x, y, z^2, yz)$. We prove that, generically, the germs of height functions defined on the 3D-crosscap have $_V\mathcal{K}$ -codimension at most 3. Then the analysis of the geometry of these function germs provides geometrical information on the crosscap points curve, the double points surface and on the contact of the 3D-crosscap with hyperplanes in \mathbb{R}^4 .

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The sequence of mixed Lojasiewicz exponents associated to pairs of ideals and log-canonical thresholds

CARLES BIVIÀ-AUSINA

Universitat Politècnica de València - Spain

We analyze the sequence $\mathcal{L}_{J}^{*}(I)$ of mixed Łojasiewicz exponents attached to any pair I, Jof monomial ideals of finite colength of the ring of analytic function germs $(\mathbb{C}^{n}, 0) \rightarrow C$. In particular, we obtain a combinatorial expression for this sequence when J is diagonal. For more general classes of pairs of ideals, we also show several relations of $\mathcal{L}_{J}^{*}(I)$ with other numerical invariants, like the log-canonical threshold lct(I) of I, and the relation of these numbers with the notion of bi-Lipschitz equivalence of ideals.

μ -constant deformations of functions on ICIS

RAFAELA SOARES DE CARVALHO, ORÉFICE OKAMOTO AND JOÃO NIVALDO TOMAZELLA

UFSCar - Brazil

Greuel, in 1986, studied properties of the constancy of the Milnor number of a deformation $f_t : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ of a holomorphic function germ with isolated singularity resulting in the following theorem:

Theorem: Let $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ be a holomorphic function germ with isolated singularity at the origin. For any deformation $F : (\mathbb{C}^n \times \mathbb{C}) \to (\mathbb{C}, 0)$ of f the following statements are equivalent:

(1) F is μ -constant.

(2) For every holomorphic curve $\gamma : (\mathbb{C}, 0) \to (\mathbb{C}^n \times \mathbb{C}, 0)$

$$\nu\left(\frac{\partial F}{\partial t}\circ\gamma\right) > \inf\{\nu\left(\frac{\partial F}{\partial x_i}\circ\gamma\right) \mid i=1,\ldots,n\},\$$

(where ν denotes the usual valuation of a complex curve). (3) Same statement as in (2) with ">" replaced by " \geq ".

(4) $\frac{\partial F}{\partial t} \in \overline{J_F}$, (where $\overline{J_F}$ denotes the integral closure of the Jacobian ideal $J_F = \left\langle \frac{\partial F}{\partial x_1}, \ldots, \frac{\partial F}{\partial x_n} \right\rangle$ in \mathcal{O}_{n+1}).

(5) $\frac{\partial F}{\partial t} \in \sqrt{J_F}$, (where $\sqrt{J_F}$ denotes the radical of J_F).

(6) $v(J_F) = \{0\} \times \mathbb{C} \text{ near } (0,0).$

In this work, we studied this result for families $f_t : (X, 0) \to (\mathbb{C}, 0)$, where (X, 0) is an ICIS.

Parallelizable Spheres and Real Projective Spaces

JOSÉ LEÔNCIO CASTELO BRANCO JÚNIOR

Universidade Federal da Paraíba - Brazil

In this work we study conditions for the sphere S^n and the real projective space $\mathbb{R}P^n$ to be parallelizable. In a parallelizable smooth manifold M it is possible to define a collection of continuous vector field. From such collections of vector fields one can make inferences about some properties of the tangent spaces T_pM with $p \in M$. However, some mathematicians have noticed that S^2 is not parallelizable. Thus, the following doubt naturally arises:

When is a smooth manifold parallelizable?

Using a tool from the Algebraic Topology, called Stiefel-Whitney classes, we will determine conditions for a sphere S^n and the space $\mathbb{R}P^n$ to be parallelizable. These two smooth manifold are the classic examples in that theory. The obtained results are that the only parallelizable spheres are S^1 , S^3 and S^7 and that the space $\mathbb{R}P^n$ can be parallelizable if n+1 is a power of 2.

Residues for maps generically transverse to foliations

Leonardo M. Câmara

UFES - Brazil

We prove the existence of residues related to tangency points of maps generically transverse to holomorphic foliations. We provide some formulas relating these residues to the characteristic classes of these foliations. Joint work with Mauricio Corrêa JR

Euler obstrucion of EIDS

NANCY CHACHAPOYAS

UNIFEI - Brazil

In this work, we study the Euler obstruction of essentially isolated determinantal singularities (EIDS). The EIDS were defined by W. Ebeling and S.M. Gusein-Zade, as a generalization of isolated singularity. Here are given some formulas to calculate the Euler obstruction for the determinantal varieties with singular set an ICIS, this results are find in CHACHAPOYAS SIESQUÉN, NANCY CAROLINA . Euler obstruction of essentially isolated determinantal singularities . TOPOLOGY AND ITS APPLICATIONS, Volume 234, Pages 166-177, 2018.

Finiteness theorems for the classification of map germs and of pairs of map germs

JOÃO C. F. COSTA

UNESP - Brazil

In this work we present several results about finiteness theorems in the classification problem of map germs and of pairs of map germs, up to variations of the contact equivalence. More precisely, we introduce the notion of bi-topological contact equivalence of pairs of map germs and the notion of multi-K-bi-Lipschitz equivalence of map germs and present the finiteness results for these equivalence relations. These results derive from joint work with L. Birbrair, E.S. Sena Filho and R. Mendes.

Affine Umbilic Points of Surfaces in 3-Space

MARCOS CRAIZER

PUC-Rio - Brazil

In this talk we discuss the behavior of the affine curvature lines close to isolated affine umbilic points of a surface in 3-space. We consider also related topics, like approximation by affine spheres and an affine Loewner's conjecture. Finally we discuss the affine cylindrical pedal in 4-space, whose inflection points correspond to the affine umbilic points of the original surface. This is a joint work with Ronaldo Garcia and Martin Barajas.

The infinitesimal Lipschitz conditions

THIAGO F. DA SILVA Universidade Federal do Espírito Santo - Brazil

Joint work with T. Gaffney

In [4], Gaffney used the integral closure of modules to describe the Whitney equisingularity. If $X \subseteq \mathcal{B}^C(\mathbb{R}^{2n})^n \times \mathcal{B}^C(\mathbb{R}^{2n})^k$ is a family of complex analytic varieties defined by an analytic map $F : (\mathcal{B}^C(\mathbb{R}^{2n})^n \times \mathcal{B}^C(\mathbb{R}^{2n})^k, 0) \to (\mathcal{B}^C(\mathbb{R}^{2n})^p, 0), Y = 0 \times \mathcal{B}^C(\mathbb{R}^{2n})^k \equiv \mathcal{B}^C(\mathbb{R}^{2n})^k \subseteq X$ the parameter space and the singular locus of a small enough representative of X, Gaffney showed that (X - Y, Y) is Whitney equisingular if and only if all the partial derivatives of Fwith respect to the parameter space are in the integral closure of the submodule generated by $\left\{z_i \frac{\partial F}{\partial z_j}\right\}_{i,j=1}^n$, where $z_1, ..., z_n$ are the coordinate functions on $\mathcal{B}^C(\mathbb{R}^{2n})^n$. We denote this inclusion as

$$JM(X)_Y \subseteq \overline{m_Y JM_z(X)}.$$

Since there is a close relation between the double structure and Lipschitz behavior, it is natural to hope that the above condition may desbribe the bi-Lipschitz equisingularity adding the double structure, i.e, $(JM(X)_Y)_D \subseteq (\overline{(m_Y JM_z(X))_D})_D$, or even a weaker condition as $(JM(X)_Y)_D \subseteq (\overline{JM_z(X)})_D$. In the hypersurface case, Gaffney [2] called these the infinitesimal Lipschitz conditions m_Y and A, respectively.

In this talk we will generalize these notions for any codimension and to present a generalization of the Genericity Theorem, proved by Gaffney in the hypersurface case, which states that the iL_A condition holds generically on the parameter space. At the end, we present an application on the Grassmanian modification of a given analytic variety.

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Line Bitangencies: Transitions and Global Theorems

DAN DREIBELBIS

University of North Florida - United States of America

Given two surfaces immersed in Euclidean 3-space, we expect a two-parameter family of line bitangencies between them. This can be considered a surface in the Cartesian product of the surfaces, and the singularity of its projection onto a surface leads to various geometric curves on the surface. Furthermore, the line bitangencies describe a direction field on the surfaces. In this talk, we describe these geometric objects, and then describe how they can generically change locally in a one-parameter family of surfaces pairs. In addition, we look at how these objects relate to each other in a global sense.

Degeneration of Torelli theorem

ANANYO DAN BCAM-Basque Centre for applied mathematics - Spain The classical Torelli theorem states that a complex, smooth, projective curve is uniquely determined by its principally polarized Jacobian variety. Mumford and Newstead further prove that the Jacobian of the curve is uniquely determined by the (second) intermediate Jacobian of the moduli space of stable rank 2 vector bundles with determinant of odd degree over the curve. However their techniques fail when the curve is singular. We use degeneration techniques from Hodge theory to prove an analogous result when the underlying curve is irreducible nodal with a single node. This is joint work with I. Kaur and S. Basu.

On the characteristic curves on surfaces in \mathbb{R}^4 .

JORGE L. DEOLINDO-SILVA Universidade Federal de Santa Catarina - Brazil

We study the geometry of the characteristic curves on surfaces in \mathbb{R}^4 . The characteristic curves are the analogy of the asymptotic curves in elliptic regions. We show the locus of inflections of the characteristic curves at a $P_3(c)$ -point has geometrical meaning on surface using the cr-invariant.

On Macaulayfication of Sheaves

DOAN TRUNG CUONG

Institute of Mathematics, Vietnam Academy of Science and Technology - Vietnam

Macaulayfication of an algebraic scheme is a weak form of desingularization where regularity is replaced by Cohen-Macaulay property. Question on the existence of Macaulayfication was asked by Faltings and Brodmann and is answered in a very general context by Kawasaki. Macaulayfication could be defined for coherent sheaf in a natural way. In this presentation, I will discuss the existence of Macaulayfication of a coherent sheaf.

The Gauss-Bonnet Theorem for coherent tangent bundle over surfaces with boundary

WOJCIECH DOMITRZ

WarsawUniversity of Technology - Poland

This is a joint work with M. Zwierzynski. In 2008 K. Saji, M. Umehara, K. Yamada proved the Gauss-Bonnet formulas for coherent tangent bundles over compact oriented surfaces (without boundary). We establish the Gauss-Bonnet theorem for coherent tangent bundles over compact oriented surfaces with boundary. We apply this theorem to investigate global properties of maps between surfaces with boundary. As a corollary of our results we obtain Fukuda-Ishikawa's theorem. We also study geometry of the affine extended wave fronts for planar closed non singular hedgehogs (rosettes). In particular, we find a link between the total

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geodesic curvature on the boundary and the total singular curvature of the affine extended wave front, which leads to a relation of integrals of functions of the width of a resette.

Topological stability and the Whitney fibering conjecture

ANDREW DU PLESSIS

Aarhus - Denmark

Recent work (together with Claudio Murolo and David Trotman) proving the Whitney fibering conjecture for C-regular stratifications allows considerable advances in the theory (described in my book with Terry Wall) characterising topological stability of smooth mappings. I will describe the relation between the fibering conjecture and topological stability, and give an example of its use in understanding the stability and versality of unfoldings of binary quintics: this will draw on some key calculations in recently published joint work with Wall).

Recent developments in singularities of general polynomial mappings

MICHAL FARNIK

Jagiellonian University - Poland

Let $F : \mathbb{C}^n \to \mathbb{C}^n$ be a general polynomial mapping of degree (d_1, \ldots, d_n) . I will examine the Thom-Boardman strata of F of codimension at most four, that is Σ^1 , $\Sigma^{1,1}$, $\Sigma^{1,1,1}$, $\Sigma^{1,1,1,1}$ and Σ^2 . In particular I will compute their degrees in terms of (d_1, \ldots, d_n) . Then I will proceed with examining the determinant of F. This is joint work with Z. Jelonek and M.A.S. Ruas.

Functions on a swallowtail

ALEX P. FRANCISCO

ICMC-USP - Brazil

We classify submersions from $(\mathbb{R}^3, 0)$ to $(\mathbb{R}, 0)$ up to diffeomorphisms which preserve the swallowtail and use this classification to study its flat geometry. The flat geometry is derived from the contact of the swallowtail with planes, which is measured by the singularities of the height function.

Desingularization of 2-dimensional schemes – from the complex to the arithmetic case

Anne Frühbis-Krüger

Hannover - Germany

Resolution of singularities in characteristic zero in arbitrary dimension has been proved by Hironaka in his monumental work in the 1960s. His result has been made constructive and algorithmic in different variants since then. In positive and mixed characteristic, however, only results in small dimensions are known. In this talk, I shall shortly review the basic ideas of characteristic zero and speak about the difficulties arising as soon as one passes beyond characteristic zero. I shall then focus on known approaches for two dimensional schemes, which carry over to mixed characteristic. In paricular, I will focus on explicit constructions that can be formulated algorithmically and allow implementation. (joint work with Bernd Schober and Lukas Ristau)

Singularities of symmetric matrices

TERENCE GAFFNEY AND MICHELLE LIRA

Northeastern - United States of America

Landscape of Symmetric Determinantal Singularities and Whitney Equisingularity.

(Joint work with Michelle Lira dos Santos Molino)

In this talk we show how the study of the Whitney equisingularity of symmetric determinantal singularities leads naturally into the study of the landscape of determinantal singularities. This study reveals the invariants which control equisingularity. In the case in which the singularity X is defined by the minors of size n-2 of a symmetric nxn matrix F, we show how to compute the invariants which arise from the study of the landscape in terms of the presentation matrix F.

Configurations in the double torus

VINICIO GÓMEZ GUTIÉRREZ

UNAM - Mexico

In this talk we present some advances of our investigation about the configurations of the ν -lines of curvature and the ν -umbilic points for some normal vector field ν with respect to some concrete embedding of the double torus in \mathbb{R}^4 . We extend our results to describe the configurations of analogous normal vector fields on other surfaces in \mathbb{R}^4 . (joint work with Federico Sánchez Bringas)

Calculating the local Euler obstruction on generic determinant varieties

NIVALDO GRULHA Universidade de São Paulo - Brazil In [4] MacPherson proved the existence and uniqueness of Chern classes for possibly singular complex algebraic varieties. The local Euler obstruction, defined by MacPherson in that paper, was one of the main ingredients in his proof.

The computation of the local Euler obstruction is not easy; various authors propose formulas which make the computation easier. For instance, Lê and Teissier provide a formula in terms of polar multiplicities [3].

In [1], Brasselet, Lê and Seade give a Lefschetz type formula for the local Euler obstruction. The formula shows that the local Euler obstruction, as a constructible function, satisfies the Euler condition relative to generic linear forms.

In order to understand these ideas better, some authors worked on some more specific situations. For example, in the special case of toric surfaces, an interesting formula for the Euler obstruction was proved by Gonzalez–Sprinberg [2], this formula was generalized by Matsui and Takeuchi for normal toric varieties [5].

A natural class of singular varieties to investigate the local Euler obstruction and the generalizations of the characteristic classes is the class of generic determinantal varieties. Roughly speaking, generic determinantal varieties are sets of matrices with a given upper bound on their ranks. Their significance comes, for instance, from the fact that many examples in algebraic geometry are of this type, such as the Segre embedding of a product of two projective spaces. Independently, in recent work [6], Zhang computed the Chern-Mather-MacPherson Class of projectivized determinantal varieties, in terms of the trace of certain matrices associated with the push forward of the MacPherson-Schwartz class of the Tjurina transform of the singularity.

We prove a surprising formula that allow us to compute the local Euler obstruction of generic determinantal varieties using only Newton binomials. Using this formula we also compute the Chern–Schwartz–MacPherson classes of such varieties.

Joint work with Terence Gaffney, Department of Mathematics, Northeastern University, Boston and Maria A. S. Ruas, Universidade de São Paulo, Instituto de Ciências Matemáticas e de Computação, Departamento de Matemática, São Carlos - SP - Brazil.

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Spectral Geometry on Stratified Spaces

LUIZ HARTMANN

Federal University of São Carlos - Brazil

In his seminal work "Spectral Geometry of Singular Riemannian Spaces", Jeff Cheeger started an influential program on spectral analysis on stratified spaces with singular Riemannian metrics. The ultimate goal of the program is to establish resolvent trace asymptotics in this very general setting. We propose to discuss the development of Cheeger's program until the recent days.

The Artin-Greenberg function for plane curve germs in arbitrary characteristic

Abramo Hefez UFF - Brazil In this talk we introduce the notion of Hamburger-Noether tree that substitutes Egger's tree in arbitrary characteristic and show how it simplifies in this context many known results. We also use it to determine the order of contact of an ideal and the Artin-Greenberg function for plane curve germs in arbitrary characteristic. This is joint work with E. M. N. de Guzmán.

Hypersurface Normalizations and Numerical Invariants

BRIAN HEPLER

Northeastern University - United States of America

We define a new perverse sheaf, the multiple-point complex, naturally associated to any "parameterized" local compete intersection, and examine how the characteristic polar multiplicities of this complex behave in certain one-parameter families of deformations of hypersurfaces with codimension-one singularities. We also discuss several interesting connections with the Milnor monodromy and mixed Hodge modules.

The image Milnor number for weigthed homogeneous map germs

IRMA PALLARÉS TORRES

BCAM - Spain

Following Ohmoto's work, we give a naive approach to obtain a explicit formula for the image Milnor number in terms of weights and degrees for weighted homogeneous map germs using known singularities. We will discuss the cases from \mathbb{C}^4 to \mathbb{C}^5 and \mathbb{C}^5 to \mathbb{C}^6 . (A joint work with Guillermo Peñafort Sanchis)

Determinacy of Determinantal Varieties

I. Ahmed and M.A.S. Ruas

COMSATS University Islamabad, Lahore Campus, Lahore - Pakistan

A more general class than complete intersection singularities is the class of determinantal singularities. They are defined by the vanishing of all the minors of a certain size of an mÃČâĂŤn-matrix. In this note, we consider G-finite determinacy of matrices defining a special class of determinantal varieties. They are called essentially isolated determinantal singularities (EIDS) and were defined by Ebeling and Gusein-Zade. In this note, we prove that matrices parametrized by generic homogeneous forms of degree d define EIDS. It follows that G-finite determinacy of matrices hold in general. As a consequence, EIDS of a given type (m,n,t) holds in general.

Topological classification of circle-valued simple Morse–Bott functions

Ingrid Sofia Meza Sarmiento

IBILCE-UNESP - Brazil

The topological classification problem of smooth functions is a classical subject in Topology and Singularity theory. However, global results and global invariants are difficult to obtain.

The main goal in this poster is to present the classification of Morse-Bott functions from S^2 to S^1 up to topological conjugacy, in the following sense: "Two Morse-Bott functions $f, g: S^2 \to S^1$ are said to be topologically conjugated if there exist homeomorphisms $h: S^2 \to S^2$ and $k: S^1 \to S^1$ such that k preserves orientation, $f = k \circ g \circ h^{-1}$ and h sends singular fibers of g to singular fibers of f".

The invariant is based on the generalized Reeb graph of the function f. Coauthors: E. B. Batista (UFCA) and J. C. F. Costa (IBILCE-UNESP)

Generic Bifurcation of reversible discontinuous piecewise smooth vector fields in 2D

IRIS O. ZELI

Unicamp - Brazil

In this work we present a study of discontinuous piecewise smooth vector fields that are reversible by a involution. We are considering here vector fields on \mathbb{R}^2 which are discontinuous on a codimension one manifold Σ , which coincides with the set of fixed points of the involution . Our goal is is to present a local study of such vector fields, describing the generic singularities

of them, when these singulaities occur in Σ .

Joint work with Marco A. Teixeira (IMECC/UNICAMP).

Topics on frontal singularities

Goo Ishikawa

Hokkaido University - Japan

The notion of frontals is a generalization of that of wave-fronts. A frontals has well-defined tangent spaces even along singular locus. I would like to talk about the characterization, the recognition and the stability of frontal singularities with related topics. Reference: Goo Ishikawa, Singularities of frontals, arXiv:1609.00488, to appear in "Singularities in Generic Geometry", Advanced Studies in Pure Mathematics, Mathematical Society of Japan.

Hamiltonian differential systems with global centers and the real Jacobian conjecture

JACKSON ITIKAWA

Universidade Federal de Rondônia - Brazil

Consider $F = (P,Q) : U \to \mathbb{R}^2$ a \mathcal{C}^2 -function, with V an open connected set in \mathbb{R}^2 , such that the Jacobian of F is nonzero. We present new classes of Hamiltonian differential systems having a global center in V.

Thom isotopy theorem for non proper maps and computation of sets of stratified generalized critical values

ZBIGNIEW TADEUSZ JELONEK

IMPAN - Poland

Let $X \subset \mathbb{C}^n$ be an affine variety and $f: X \to \mathbb{C}^m$ be the restriction to X of a polynomial map $\mathbb{C}^n \to \mathbb{C}^m$. In this paper, we construct an affine Whitney stratification of X. The set K(f) of stratified generalized critical values of f can be also computed. We show that K(f) is a nowhere dense subset of \mathbb{C}^m , which contains the set B(f) of bifurcation values of f by proving a version of the isotopy lemma for non-proper polynomial maps on singular varieties.

(coauthor is Tiep Si Dinh)

The Special Closure of Polynomial Maps and Global Non-degeneracy

JORGE A. C. HUARCAYA AND CARLES BIVIÀ-AUSINA

Universidad San Ignacio de Loyola - Peru

Let $F : \mathbb{C}^n \to \mathbb{C}^n$ be a polynomial map such that $F^{-1}(0)$ is finite. We analyze the connections between the multiplicity of F, the Newton polyhedron of F and the set of special monomials with respect to F, which is a notion motivated by the integral closure of ideals in the ring of analytic function germs $(\mathbb{C}^n, 0) \to \mathbb{C}$. In particular, we characterize the polynomial maps whose set of special monomials is maximal.

Multiple points of a simplicial map and the Image Computing Spectral Sequence

JOSÉ LUIS CISNEROS MOLINA

UNAM - Mexico

Joint work with David Mond

The Image-Computing Spectral Sequence computes the homology of the image of a finite map from the alternating homology of the multiple point spaces of the map. In this talk we present a new proof of this, in case the map can be triangulated. Thanks to work of Hardt, this holds for a very wide range of maps, and in particular for most of the finite maps of interest in singularity theory. The proof seems simpler and more canonical than earlier proofs.

Double points in families of map germs from \mathbb{R}^2 to \mathbb{R}^3 an \mathcal{C}^0 -finite determinacy

JUAN ANTONIO MOYA PÉREZ

Universitat de València - Spain

This talk is divided in two parts: In the first part we show that a 1-parameter family of real analytic map germs $f_t : (\mathbb{R}^2, 0) \to (\mathbb{R}^3, 0)$ with isolated instability is topologically trivial (has constant topological type) if it is excellent and the family of double point curves $D(f_t)$ in $(\mathbb{R}^2, 0)$ is topologically trivial (has constant topological type). In particular, we deduce that f_t is topologically trivial when the Milnor number $\mu(D(f_t))$ is constant.

In the final part, we show, as an application of this results, that any real analytic map germ $f(\mathbb{R}^2, 0) \rightarrow (\mathbb{R}^3, 0)$ with isolated singularity is \mathcal{C}_0 -finitely determined. (joint work with Juan José Nuño Ballesteros)

SO(3) normal forms and differential geometric invariants of singular surfaces

Kentaro Saji

Kobe University - Japan

In this talk, we deal with "SO(3) normal forms" of singular surfaces. Let us take an element $f \in C^{\infty}(2,3)/\sim_{\mathcal{A}}$. We consider a "simplest" representative with respect to the action of $\text{Diff}(2) \times SO(3)$ to $C^{\infty}(2,3)/\sim_{\mathcal{A}}$ (called an SO(3) normal form of f in this talk). Since SO(3) is quite smaller than Diff(3), some modulus remain, and they are differential geometric (SO(3)) invariants. This kind of normal form for regular surfaces is called the Monge form and for cross cap, it is obtained by Bruce and West. This form has been used for studying differential geometric invariants of cross cap. In this talk, several SO(3) normal forms for other singularities will be given. Furthermore, applications of them will be also given.

Double of stable singularity on boundary

ΚΑΖUΤΟ ΤΑΚΑΟ

Kyoto University - Japan

It is usual in topology to consider gluing manifolds along their boundaries, together with something defined on the manifolds. In particular, taking the "double" of given one, namely gluing it and its mirror image, is typical. I consider taking the "double" of a 3-manifold with boundary, together with a smooth map into a 2-manifold without boundary. It might be naively hoped that, if the original map has a stable singularity on the boundary, then the doubled singularity is stable. In this talk, I justify it for some types of stable singularities, but deny it for the other types.

Konstantinos Kourliouros

ICMC-USP - Brazil

A section of a Hamiltonian system is a hypersurface in the phase space of the system, usually representing a set of one-sided constraints. We present local classification results for generic singularities of such sections and of their associated Hamiltonian systems with constraints. In particular we obtain exact normal forms with functional invariants (moduli) and we show how these invariants are related with the action integrals of the Liouville 1-form associated to the symplectic form, along certain integral curves of the Hamiltonian vector field. The latter generalise, in the symplectic setting, the vanishing half-cycles of boundary singularities and of projections of plane curves onto the line.

Exponential convexifying of polynomials

KRZYSZTOF KURDYKA

Université Savoie Mont Blanc - France

Let $X \subset \mathbb{R}^n$ be a convex closed and semialgebraic set and let f be a polynomial positive on X. We prove that there exists an exponent $N \geq 1$, such that for any $\xi \in \mathbb{R}^n$ the function $\varphi_N(x) = e^{N|x-\xi|^2}f(x)$ is strongly convex on X. When X is unbounded we have to assume also that the leading form of f is positive in $\mathbb{R}^n \setminus \{0\}$. We obtain strong convexity of $\Phi_N(x) = e^{e^{N|x|^2}}f(x)$ on possibly unbounded X, provided N is sufficiently large, assuming only that f is positive on X. We apply these results for searching critical points of polynomials on convex closed semialgebraic sets. Joint work with Katarzyna Kuta and Stanisław Spodzieja.

Linear equations on real algebraic surfaces

WOJCIECH KUCHARZ

Jagiellonian University - Poland

We prove that if a linear equation, whose coefficients are continuous rational functions on a nonsingular real algebraic surface, has a continuous solution, then it also has a continuous rational solution. This is known to fail in higher dimensions. Joint work with Krzysztof Kurdyka.

Homogeneous Polynomials, a General Point of View

LEON KUSHNER

UNAM - Mexico

Homogeneous Polynomials, a General Point of View Leon Kushner

In this talk, the first part is a review of unique factorization domains, in particular, the ring of polynomials in n-variables.

For a partial classification of $H_n(2)$, the ring of homogeneous polynomials of degree n in two variables, it is noted that three linearly independent lines define a unique isomorphism of the plane sending them to the x-axis, y-axis and x + y = 0 respectively.

We then consider an irreducible quadratic factor times a linear factor and the case of the product of independent quadratic factors and their canonical form.

For the product of four and five linearly independent linear factors. For the first case we use the cross-ratio and calculate the stabilizers and when two canonical forms are equivalent (under linear isomorphisms), in these cases appears the dihedral group (n = 4).

Finally, we finish with the case for powers of four linearly independent linear factors and the case $\lambda = -\frac{1}{2}$, the stabilizer is $D_8 \times S_3$.

Extrinsic geometry of a family of local surfaces in \mathbb{R}^4 with an isolated *v*-umbilic point of index 3/2

Lucía Ivonne Hernández Martínez

Universidad Autónoma de la Ciudad de México - Mexico

We define a family of 4 parameters of local surfaces in \mathbb{R}^4 , such that each element has a normal vector field v with an isolated v-umbilic point whose v-lines of curvature are defined by a perturbation of a quadratic form of index 3/2. We obtain the second order invariants and analyze the relation of them with the v-principal configuration. The approach we apply allow us to extend the analysis for the cases of higher index. Joint work with F. Sánchez Bringas

Invariants of first order partial differential equations

LIZANDRO SANCHEZ CHALLAPA

UFPB - Brazil

In this paper we introduce the concepts of multiplicity and index of first order partial differential equations. In particular, the concept of multiplicity coincides with the multiplicity of implicit differential equations given by Bruce and Tari in [2]. We also show that these concepts are invariants by smooth equivalences. Following the work [10] on implicit differential equations with first integrals, we introduce a definition of multiplicity for this class of equations.

Lenses on very curved zones of a singular foliation of \mathbb{C}^3

Rémi Langevin

Université de Bourgogne - France

We renormalize, using suitable lenses, small domains of a singular holomorphic foliations of \mathbb{C}^3 where the curvature is concentrated. At a proper scale, the leaves are almost translates

of a surface that we will call *profile*. A generalisation to a plane-field of the tangent cone of a singular hypersurface is defined. When the singular rays of the tangent cone are isolated, we obtain more precise results.

Cancellation of Cone-type singularities of GS-flows

DAHISY LIMA

Unicamp - Brazil

One considers a Gutierrez-Sotomayor flow φ^{GS} on a singular 2-manifold M containing regular and cone-type singularities and a cancellation theory of singular points is established. This is done by, firstly, describing the qualitative aspects of the flow φ^{GS} in terms of a chain complex $(C_*(GS), \Delta^{GS})$ generated by the singularities of φ^{GS} and whose differential "counts with sign" flow lines between them. Then, using spectral sequences analysis, one proves a topological continuation of the flow by canceling pairs of singularities associated the differentials of the spectral sequence. Furthermore, this process leads us to a minimal Gutierrez-Sotomayor flow on M, up to homotopy, containing super-attractor and super-repeller singularities. The novelty here is that M contains a singular part, hence the classical Morse Theory does not apply to this setting. To overcome this problem, one considers a Morsification of the GS-flow φ^{GS} and the smoothing of M using Lyapunov graphs as guide.

It is a joint work with Ketty A. de Rezende and Stephanie A. Raminelli.

Global Phase Portraits of the Riccati Quadratic Differential System

Bruno Domiciano Lopes

Unicamp - Brazil

In this paper we do a global analysis of the dynamics of the Riccati quadratic differential system

$$\dot{x} = 2(x), \quad \dot{y} = ky^2 + 1(x)y + 2(x)$$

with $(x, y) \in \mathbb{R}^2$, $_2(x)$ non-zero (otherwise the system is a Bernoulli differential system), $k \neq 0$ (otherwise the system is a Lienard differential system), $_1(x)$ a polynomial of degree at most 1, $_2(x)$ and $_2(x)$ polynomials of degree at most 2, and the maximum of the degrees of $_2(x)$ and $ky^2 + _1(x)y + _2(x)$ is 2. We give the complete description of its dynamics in the Poincaré disk (i.e. in the compactification of \mathbb{R}^2 with the circle \mathbb{S}^1 of the infinity) modulo topological equivalence.

Singular surfaces of revolution with prescribed unbounded mean curvature

LUCIANA F. MARTINS

IBILCE-UNESP - Brazil

We give an explicit formula for singular surfaces of revolution with prescribed unbounded mean curvature. Using it, we give conditions for singularities of that surfaces. Periodicity of that surface is also discussed. Joint work with Kentaro Saji, Samuel P. dos Santos and Keisuke Teramoto.

Milnor fibrations for certain families of mixed polynomials

RAFAELLA DE SOUZA MARTINS

Universidade Federal de Santa Catarina - Brazil

In [4] the authors started with the study of the existence of Milnor and Milnor-Lê fibrations of the following real analytic functions

$$\psi_{F,X}(z) := \langle F(z), X(z) \rangle = \sum_{j=1}^{n} F_j(z) \overline{X_j(z)} = \sum_{j=1}^{n} k_j \overline{t_j} z_{\sigma_j}^{a_j} \overline{z_j}^{b_j},$$

with $F = (k_1 z_{\sigma_1}^{a_1}, \dots, k_n z_{\sigma_n}^{a_n})$ and $X = (t_1 z_1^{b_1}, \dots, t_n z_n^{b_n})$ holomorphic vector fields, where $k_j, t_j \in \mathbb{C}^*$, $a_j, b_j \in \mathbb{Z}^+$, $j = 1, \dots, n$ and $\{\sigma_1, \dots, \sigma_n\}$ is a permutation of the set $\{1, \dots, n\}$. When this family of mixed weighted homogeneous polynomials has isolated singularity, by [4], we can assume the existence of Milnor and Milnor-Lê fibrations.

We work with a class of real analytical functions with non-isolated critical value, these are of the form

$$\psi_{F,X}(z) := \sum_{j=1}^n k_j \overline{t_j} z_j^{a_j} \overline{z_j}^{b_j},$$

with $a_j = b_j$ for at least one j and $a_j \neq b_j$ for at least one j. This functions does not satisfy the conditions of [4].We conclude the existence of Milnor [3] and Milnor-Lê [2] fibrations, using some results of the preprint [1].

We show some characteristics of this family to confirm the existence of the fiber over the regular values and the critical values and the relation between them. Its possible to made the existence of a locally trivial bundle on the discriminant of $\psi_{F,X}$ and compare the bundles for a particular case of this family. The $\psi_{F,X}$ possesses interesting properties, a particular case of which is ICIS and $\psi_{F,X}^{-1}(d), d \in \Delta_{\psi_{F,X}}$ are regular manifold.

This research was developed with Nivaldo Grulha and José Seade.

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Hypersurfaces with light-like points in Lorentzian manifolds

Masaaki Umehara

Tokyo Institute of Technology - Japan

This is a joint work with Kotaro Yamada. Recently several concrete examples of zero mean curvature surfaces in the Lorentz-Minkowski 3-space \mathbf{R}_1^3 containing a light-like line were constructed by several geometers. Here we mention the duality of singular points on maximal surfaces and explain that the fold singular points are the dual of cone-like singular points. It is well-known that maximal surfaces have real analytic extension along fold singularities.

We next consider an immersion $F : U \to \mathbf{R}_1^3$ where U is a domain of \mathbf{R}^2 . A point $o \in U$ is called a light-like point if the first fundamental form ds^2 of F is degenerate at o. We denote by B_F the determinant function of the symmetric matrix associated to ds^2 with respect to the canonical coordinate system of U. A light-like point o is said to be non-degenerate (resp. degenerate) if the gradient vector of B_F does not vanish (resp. vanishes) at o. Non-degenerate light-like points are closely related to fold singular points, and their behavior is now well-understood.

On the other hand, Klyachin proved a line theorem, that is, if o is a degenerate light-like point, then the image of F contains a light-like line segment of \mathbf{R}_1^3 passing through F(o). In this talk, we give a new simple proof of this fact, and also give a generalization of this line theorem for zero mean curvature hypersurfaces in an arbitrary Lorentzian manifold. Several related results are also given.

Comparison of the constant sheaf and intersection cohomology on a hypersurface

DAVID B. MASSEY

Northeastern University - United States of America

In the Abelian category of perverse sheaves on a complex analytic space, there is a natural surjection from the shifted constant sheaf on a hypersurface to the intersection cohomology complex. We call the kernel of this morphism the "comparison complex". We show that the comparison complex is isomorphic to the kernel of the identity minus the monodromy on the sheaf of vanishing cycles along the defining function of the hypersurface.

Lipschitz contact equivalence and analytic functions germs in \mathbb{R}^2

RODRIGO MENDES

Unilab - Brazil

The Finiteness of the Lipschitz types of real polynomial functions germs (with bounded degree) with respect to Lipschitz contact equivalence was obtained by Birbrair, Costa, Fernandes and Ruas. Later on, Ruas and Valette showed this property for map germs. At this point, the classification problem with respect to the Lipschitz contact equivalence was becomes very interesting. For subanalytic function germs in \mathbb{R}^2 , the complete classification was obtained by Birbrair, Fernandes, Gabrielov and Grandjean. For this, the authors considered so-called the pizza of a function. This object is, essentially, caracterized by a width function that is defined from the zones of arcs and your orders when composed with a given subanalytic function. Our main result is that the maximal zones of constant order obtained from such width functions has uniform size in the analytic configuration. From this, we show that the pizzas of analytic functions have special properties. Joint work with Lev Birbrair

Invariants of generic normal surface singularities

András Némethi

Renyi Institute of Mathematics - Hungary

For fixed topological type of a normal surface singularity (when the graph is a tree of rational curves) I define the notion of 'generic' analytic structure. For such generic singularity one determines topologically (from the graph) the geometric genus, the cohomologies of all natural line bundles, the analytic semigroup (of divisors of natural line bundles), the multiplicity, etc. The proofs are based on a new package of techniques and results developed for surface singularities. All the results are joint work with my student J. Nagy.

Filipe Bellio da Nóbrega

PUC-Rio - Brazil

The aim of this work is to study the topological and geometric properties of closed generic oriented immersed curves $i : \mathbb{S}^1 \to \mathbb{R}^2$ in the plane. In this case, generic means that the curve can only have double points without a common tangent. One can label the singularities using n symbols, such as $a_1, ..., a_n$. Going around the curve, a cyclic word of length 2n is produced. However, not every word is related to a planar curve, there are requirements on its combinatorics, the first of which was found by Gauss. Advances were made in the study of locally convex curves on the plane, the sphere and the projective plane.

Geometry of singularities of a Pinchuk's map

NGUYEN THI BICH THUY

IBILCE UNESP - Brazil

We determine explicitly a Valette variety associated to the Pinchuk's map of smallest degree and calculate its intersection homology. That provides geometries of singularities of this Pinchuk's map. Moreover, the result provides a counter-example for the "real version" of Anna and Guillaume Valette's result.

(The work is available on: https://arxiv.org/pdf/1710.03318.pdf)

New trends on singularities of mappings

JUAN J. NUÑO-BALLESTEROS

Universitat de València - Spain

In this talk, we will focus on the following aspects of the theory of singularities of mappings:

1) Topological classification of singularities of real analytic map germs. Given a real analytic map germ $f: (\mathbb{R}^n, 0) \to (\mathbb{R}^p, 0)$ with isolated instability, the C^0 - \mathcal{A} -class of f is determined by its link, obtained by intersecting the image of f with a sphere S_{ϵ}^{p-1} centered at the origin in \mathbb{R}^p , with $\epsilon > 0$ small enough. If $f^{-1}(0) = \{0\}$ (which always happens when $n \leq p$), the link is a C^0 -stable mapping $\gamma: S^{n-1} \to S^{p-1}$ and f is C^0 - \mathcal{A} -equivalent to the cone of its link. The case of $f^{-1}(0) \neq \{0\}$ is more complicated, since in this case the link is a C^0 -stable mapping $\gamma: N \to S^{p-1}$, where N is a manifold with boundary and instead of the usual cone we need to use a generalized version of it. We will review some known results for small values of n and p and discuss about open problems: topological triviality of families, C^0 -finite determinacy, etc.

2) Holomorphic map germs defined over singular varieties. We consider a holomorphic map germ $f : (X,0) \rightarrow (\mathbb{C}^p,0)$, where (X,0) is a variety with isolated singularity and f has isolated instability. We are interested in what happens when deforming the variety and the map simultaneously. The most convenient case is when (X,0) is an ICIS (isolated complete intersection singularity), since Mond and Montaldi developed the basic machinery for

the infinitesimal approach (stability, versality, etc). We look for geometrical invariants which extend the known invariants in the smooth case and try to generalize some known results. We will show some results in the case of curves and surfaces and discuss about the open problems.

3) Singularities of mappings at infinity. The study of singularities at infinity has been developed for polynomial functions $f : \mathbb{C}^n \to \mathbb{C}$. Following Broughton, a polynomial function $f : \mathbb{C}^n \to \mathbb{C}$ is tame if it has isolated critical points and it has no critical values coming from infinity. This allows to obtain a global Milnor fibration and to study the topology of the generic fibre. In recent works, Farnik, Jelonek and Ruas also explore global properties and invariants of polynomial mappings $f : \mathbb{C}^2 \to \mathbb{C}^2$. We will propose a definition of "tame" for general polynomial mappings $f : \mathbb{C}^n \to \mathbb{C}^p$ and discuss about open problems.

Geometry of Singular Surfaces from viewpoint of contact theory

TAWANA G. NUNEZ AND LUCIANA F. MARTINS

Unesp - Brazil

The geometry of singular surfaces can be considered using tools of Singularity Theory. More precisely, on studying the singularities of two special functions: given a manifold $M \subset \mathbb{R}^n$, the height function on M, $h_v: M \to \mathbb{R}$, given by $h_v(x) = \langle x, v \rangle$, which measures the contact with planes and the squared functions on M, $d_a: M \to \mathbb{R}$, given by $d_a(x) = |a - x|^2$, which measures the contact with spheres. In this work, we study the contact between a corank 1 surface of \mathbb{R}^3 and planes and spheres. For this, it is necessary to study the basic theory for these surfaces, i. e., its tangent space and normal, the fundamental forms, asymptotic directions and the definition of a special curvature called umbilic curvature. Besides we also study special points on manifolds called Roundings and Flattenings, which are defined using the singularities of the previously functions by viewpoint of the Boardman symbols. Again, we focus on results for surfaces with singularities. This is part of the master dissertation of the author under supervision of Luciana F. Martins and it is based on the following papers: Isolated Roundings and Flattenings of Manifolds in Euclidean Spaces of T. Fukui and J.J. Nuño-Ballesteros (Tohoku Math J., 2, 57 ,2005, 469-503) and Contact properties of surfaces in \mathbb{R}^3 with corank 1 singularities of L.F. Martins and J.J. Nuño-Ballesteros (Tohoku Math J., 67, 2015, 105-124).

Invariant theory for the Lorentz group on the Minkowski space

LEANDRO NERY DE OLIVEIRA, MIRIAM GARCIA MANOEL Universidade Federal do Acre - Brazil

We introduces the study of applications defined on the Minkowski space with symmetry. We adapted some known results from the classical theory to the Lorentz group acting on the Minkowski space in order to find the equivariant maps from the invariant functions in the cartesian, with diagonal action, and vice versa. We present an algorithm for calculating a Hilbert base of the ring of the invariant polynomials of a Γ group once we know the Hilbert

base of the ring of the invariant polynomials of a subgroup of Γ of even index, using the Reynolds operators.

The extra-nice dimensions

RAÚL OSET SINHA Unviersitat de València - Spain

We define the extra-nice dimensions and prove that the subset of locally stable 1-parameter families in $C^{\infty}(N \times [0,1], P)$ is dense if and only if the pair of dimensions $(\dim N, \dim P)$ is in the extra-nice dimensions. This result is parallel to Mather's characterization of the nice dimensions as the pairs (n, p) for which stable maps are dense. The extra-nice dimensions are characterized by the property that discriminants of stable germs in one dimension higher have \mathcal{A}_e -codimension 1 hyperplane sections. They are also related to the simplicity of \mathcal{A}_{e^-} codimension 2 germs. We give a sufficient condition for any A_e -codimension 2 germ to be simple and give an example of a corank 2 codimension 2 germ in the nice dimensions which is not simple. Then we establish the boundary of the extra-nice dimensions. Finally we answer a question posed by Wall about the codimension of non-simple maps. Joint work with M. A. S. Ruas and R. Wik Atique

On complete reducibility and étale slice theorem in characteristic p > 0.

ARYAMPILLY JAYANTHAN PARAMESWARAN

Tata Institute of Fundamental Research - India

Let G be a reductive group over a field K which is algebraically closed of characteristic p > 0. We prove a structure theorem for a class of subgroup schemes of G when p is bounded below by the Coxeter number of G. As applications, we derive semi-simplicity results generalizing earlier results of Serre proven in 1998 and also obtain an analogue of Luna's étale slice theorem for suitable bounds on p. Jointly with V. Balaji, P. Deligne.

Curve monodromy, quasi-periodic diffeomorphisms and tête-à-tête graphs.

Maria Pe Pereira

Universidad Complutense de Madrid - Spain

I will report about a joint work with J. Fernández de Bobadilla and P. Portilla which is also part of the PhD Thesis of the third author.

Norbert A'Campo defined tête-à-tête graphs and showed that if the monodromy of a plane branch is periodic then it is a generalized Dehn twist along a tête-à-tête graph.

We see that any periodic orientable diffeomorphisms of surfaces with non-empty boundary is induced by a generalized Dehn twist along a tête-à-tête graph. In this sense we generalize a result by Christian Graf.

We also study the more general case of quasi-periodic homeomorphisms of surfaces with boundary. Monodromy of an arbitrary plane curve is an example of them. To codify this type of homeomorphisms we introduce the notion of mixed tête-à-tête graph, improving a former version by A'Campo. We show that any quasi periodic homeomorphism, subject to certain combinatorial restriction, can be modelized with a mixed tête-à-tête graph. These restrictions are accomplished by the monodromy of unibranch plane curves.

In this talk I will introduce the monodromy of plane branches, the quasi-periodic automorphisms, the tête-à-tête graphs and the mentioned characterizations.

Iterated Multiple Points and A-finiteness for maps of any corank

Guillermo Peñafort

Basque Center For Applied Mathematics - Spain

Since their introduction by Kleiman in the 80s, the iterated multiple point spaces of a holomorphic map have only been well understood in the absence of points of corank greater than one. We overcome the technical difficulties associated to the corank with the introduction of multiple point functors. This rather abstract approach gives explicit equations for multiple point spaces with no hypothesis on the corank. We also describe some pathological lack of symmetry and excess of dimension. As a consequence, we obtain a computable criterion of A-finite determinacy for map-germs of any corank in a wide range of source and target dimensions. This is joint work with Juan José Nuño Ballesteros.

Characterizing Lipschitz Normal Embedding of Surface Singularities

Helge Møller Pedersen

UFC - Brazil

Any real or complex singularity (X, 0) is equipped with two natural metrics. The *outer metric*, which is the restriction of the ambient euclidian metric, and the *inner metric*, which is the metric associated with an riemannian metric on the germ. Up to bilipschitz equivalence these metrics does not depends on the choices of embedding. The inner and outer metrics are in general not bilipschitz equivalent, and one says that (X, 0) is *Lipschitz normally embedded* if they are bilipschitz equivalent. In a recent paper Birbrair and Mendes gave an *arch criterion* for checking whether a singularity is Lipschitz normally embedded by testing pairs of real arcs. In this talk we will restrict to normal complex surface singularities. We will then show several new version of the arc criterion, reducing greatly the number of curves one need to check, and finally a version using complex arcs instead of real arcs.

On the Milnor classes of local complete intersections

M.F.Z.Morgado

IBILCE - São José do Rio Preto

In this work we study algebraic, geometric and topological properties of the Milnor classes of local complete intersections with arbitrary singularities. We describe first the total Milnor class of the intersection of a finite number of hypersurfaces, under certain conditions of transversality, in terms of the Milnor classes of the hypersurfaces. Using this description we obtain a Parusinski-Pragacz type formula, an Aluffi type formula and a description of the Milnor class of the local complete intersection in terms of the global Lê classes of the hypersurfaces that define it.

Dynamical Cancellation of Double Crossing Singularities via Spectral Sequences

Stephanie A. Raminelli

Unicamp - Brazil

The goal of this work is to obtain dynamical information from algebraic-topological tools found in Conley Theory and used to explore filtered chain complexes and their underlying spectral sequences. More specifically, we will approach the concepts of Morsification and chain complex associated to a Gutierrez-Sotomayor flow containing only regular and double crossing singularities. Considering a spectral sequence (E^r, d^r) associated to this complex, we observe algebraic cancellations of the modules E^r determined by the nonzero differentials d^r 's.

These algebraic cancellations are dynamically interpreted as a history of birth and death of connecting orbits of the flow caused by the cancellation of consecutive singularities. Furthermore, we construct a family of Gutierrez-Sotomayor flows associated to the spectral sequence which also defines a continuation to the minimal flow on the singular manifold M.

It is a joint work with Ketty A. de Rezende and Dahisy Lima.

Local volumes and singularities

Antoni Rangachev

University of Chicago - United States of America

In this talk I will discuss some exciting new connections between the local volume, an invariant studied in binational geometry, and the deformation theory of singularities. I will introduce a class of singularities that generalizes the class of smoothable singularities: these are all singularities that admit deformations to singularities with deficient conormal spaces. I will discuss how this new class arises from problems in differential equisingularity and how it relates to the local volume of a line bundle.

A survey on structurally unstable quadratic differential systems of codimension two in the plane

ALEX C. REZENDE

UFSM - Brazil

In the attempt of classifying all the quadratic differential systems, J.C. Artés, R. Kooij, J. Llibre and A.C. Rezende have performed a systematic analysis (modulo limit cycles) of such differential systems, starting from the most generic ones, according to their structural stability (codimension zero), to the most degenerate ones (codimension one, two, and so on); until now, the concluded cases are the families of codimension-zero and codimension-one systems. In this work we present an outline on the specific family of structurally unstable quadratic differential systems of codimension-two in the plane. We summary the main features of codimension-zero and codimension-zero and codimension-zero for codimension-zero and codimension-two quadratic differential systems, and we emphasize the characteristics of codimension-two quadratic systems, showing some predictions of what we might find out by the end of the study of this case. This is a joint work with Joan C. Artés, Jaume Llibre and Regilene D.S. Oliveira.

Milnor-Hamm fibrations for real singularities

RIBEIRO, M.; ARAÚJO DOS SANTOS, R. N.; TIBAR, M. Universidade Federal do Espírito Santo - Brazil

We introduce two local fibration structures defined by a map germ $G : (\mathbb{R}^m, 0) \to (\mathbb{R}^p, 0)$, $m \ge p > 1$, motivated by the emergence of new classes of real maps. The first fibration deals with the complementary of the discriminant and we call it Milnor-Hamm fibration, whereas the second type called Singular Milnor fibration, takes into account the fibres over the stratified discriminant.

We give conditions under which a locally trivial fibration over $\mathbb{R}^p G$ exists. We next introduce the ∂ -Thom regularity, a weaker Thom condition adapted to our setting and prove that it is sufficient for the existence of Milnor-Hamm fibrations. We find classes of singularities which satisfy our conditions, for instance certain maps of type $f\bar{g}$. We also show examples where the map G has Milnor-Hamm fibration without being ∂ -Thom regular, which is also a new feature.

Next, we introduce the *singular Milnor tube fibration* which includes the fibres over the discriminant. We give an existence condition which is more general than through the Thom regularity, and give examples of singular tube fibrations with and without Thom regularity.

0-Stable singularities of corank one quasi homogeneous map germs from \mathbb{C}^n to \mathbb{C}^p , $n \ge p$.

E. C. RIZZIOLLI UNESP-Rio Claro-SP

This work was done jointly with Prof.Dr Marcelo José Saia-ICMC-USP and Prof. Dr Aldicio José Miranda-FAMAT-UFU and deals with the geometry of quasi homogeneous co-rank one

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finitely determined map germs from $(\mathbf{C}^n, 0)$ to $(\mathbf{C}^p, 0)$ with $n \ge p$. The class of invariants which we study are the isolated singularities, called 0-stable singularities because they are the 0-dimensional singularities. To get such formula we apply the Hilbert's Syzygy Theorem to determine the graded free resolution given by the Syzygy modules of the associated iterated Jacobian ideal.

A weak version of Mond's Conjecture for corank 1 maps and applications

Roberto Giménez Conejero

Universitat de València - Spain

In this work we have proved that, being $f : (\mathbb{C}^n, S) \to (\mathbb{C}^{n+1}, 0)$ of corank 1 and finitely *A*-determined, $\mu_I(f) = 0$ if, and only if, f is a stable germ. This is a weak version of the Mond's Conjecture, $\mu_I(f) \ge_{A_e} (f)$, with our hypotheses.

We also develop some interesting results, for example the upper semicontinuity of μ_I , to prove a Houston's conjecture on excellent unfoldings in Gaffney's sense. The formal statement of this conjecture is that f_t is excellent if, and only if, $\mu_I(f_t)$ is constant, for corank 1 germs.

Invariants of Parametrized Monomial Surfaces in 4-space

MARIA ELENICE RODRIGUES HERNANDES

Universidade de Maringá - Brazil

In this work we classify germs of parametrized monomial surfaces defined by $f : (\mathbb{C}^2, 0) \rightarrow (\mathbb{C}^4, 0)$ that are \mathcal{A} -finitely determined, where \mathcal{A} is the group of right-left equivalences. More precisely we prove that a parametrized monomial map f is \mathcal{A} -finitely determined if and only if f is an immersion or f is \mathcal{A} -equivalent to $(x, x^\lambda y, y^n, y^m)$ with gcd(n, m) = 1. The semigroup of values Γ of the parametrized plane curve (y^n, y^m) associated to f(0, y), is the main tool in order to obtain the classification and some formulae or estimates of invariants as the delta invariant δ_f , the number of double points d(f) appearing in a stable perturbation of f and the \mathcal{A}_e -codimension of f. Our focus is the computation of δ_f , defined as the complex dimension of $\mathcal{O}_2/f^*(\mathcal{O}_4)$. We prove that δ_f can be calculated in terms of λ , the complement of semigroup $\mathbb{N} \setminus \Gamma$ and of the delta invariant of the parametric curve $\phi(y) = (y^n, y^m)$. It is a well known result that the number of double points appearing in a stable perturbation of a plane curve coincide with the delta invariant of the curve. This is not true in our case, but we obtain that $d(f) \leq \delta_f$ with equality if and only if n = 2.

This is a joint work with Maria Aparecida Soares Ruas.

Branching of limit cycles in piecewise differential systems JAUME LLIBRE, DOUGLAS NOVAES AND CAMILA RODRIGUES ICMC-Usp - Brazil Consider a differential system on the form

$$x' = F_0(t, x) + \sum_{i=1}^k \varepsilon^i F_i(t, x) + \varepsilon^{k+1} R(t, x, \varepsilon),$$

where $F_i: \mathbb{S}^1 \times D \to \mathbb{R}^m$ and $R: \mathbb{S}^1 \times D \times (-\varepsilon_0, \varepsilon_0) \to \mathbb{R}^m$ are piecewise C^{k+1} functions and T-periodic in the variable t. Assuming that the unperturbed system $x' = F_0(t, x)$ has a d-dimensional submanifold of periodic solutions with d < m we use the Lyapunov-Schmidt reduction method and the averaging theory to study the existence of limit cycles of the above differential system.

Singularities of secant maps, local and global

MARIA C. ROMERO FUSTER

Valencia - Spain

The local properties of secant maps associated to curves in 3-space were first analyzed by J.W. Bruce [1] who proved that, for generic pairs of curves, such maps are locally stable. We shall analyse the case of closed plane curves, describing its singularities from the local and multi-local viewpoints and providing their geometrical characterization up to codimension 2. We shall see that the secant map of a generic pair of closed plane curves is a stable map from the torus to the plane. From a global viewpoint, we determine the number of connected compo- nents of the singular set of the secant map of a generic pair of closed convex curves in terms of their respective Whitney indexes. We also describe the non convex case ([2])

On the other hand, the image of the singular set of the secant map is a collection of closed plane curves with isolated cusps corresponding to pairs of points having parallel tangents and equal (non vanishing) curvature. We call it the secant caustic. We shall comment some global results, obtained in collaboration with W. Domitrz and M. Zwierzynski, concerning this set. We will also describe some global properties of the secant maps of curves embedded in 3-space.

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Affine curvature lines

Ronaldo Garcia UFG - Brazil

In this talk it will be considered the affine principal lines of surfaces in 3-space. We consider the binary differential equation of the affine curvature lines and obtain the topological models of these curves near parabolic points. Also it will be described the generic behavior of affine curvature lines near closed affine curvature lines. Joint work with Marcos Craizer and Martin Barajas.

Singular locus and regular fibers, do they link each other? OSAMU SAEKI

Kyushu University - Japan

Given a link L in a closed oriented 3-manifold M, we determine those links in $M \setminus L$ which can be realized as the singular point set of a generic map $M \to \mathbb{R}^2$ that has L as a regular fiber. Then, we study the linking behavior between the singular point set and regular fibers for generic maps of M into \mathbb{R}^2 .

Geometry of Focal Set

SAMUEL PAULINO DOS SANTOS

Unesp - Brazil

Samuel Paulino dos Santos and Luciana F. Martins (IBILCE - UNESP)

Let S be a regular surface without parabolic points. The focal set of S is the set:

$$F_s = F_1 \cup F_2,$$

where $F_i = \{p + \frac{1}{\kappa_i}N(p)\}, i = 1, 2, \kappa_i$ being the principal curvatures of S, and N(p) is the normal vetor to S at p. It can be shown that the focal set is the bifurcation set of the distance square function family $D: S \times \mathbb{R}^3 \to \mathbb{R}^3$, given by $D(q, a) = ||q - a||^2$. Away umbilic points, the focal set has two disconnected sheets. At umbilic points, however, the two sheets have intersection. Firstly we analyze types of singularities that can appear at the focal set, which are A_2, A_3, A_4, D_4^- e D_4^+ , being the last two appearing only at umblic points. The focal set is a surface, but it can be not regular. The points on S corresponding to singular points are called ridge. The ridge is a curve consisting of the points on S corresponding to parabolic points of focal surface are called sub-parabolics. The sub-parabolic set is the locus of geodesic inflexion of the curvature lines of S. Finally, the focal surface can be parametrized by a wave front and we can use the known results of such applications in the study of the focal set.

In short, we get geometric information about S from its focal set and vice versa. This analyses is done in some especific points, for instance, when a ridge is tangent to a curvature line.

In this poster we will go throug some of results that are in the master dissertation of the first author under supervision of Luciana F. Martins. The book *Differential Geometry from a Singularity Theory Viewpoint*, World Scientific, 2015, of S. Izumiya, M.C. Romero-Fuster, M.A.S. Ruas and F. Tari is our main reference.

The Geometry of Surfaces and the G-deformations in $\mathbb{R}^n, n = 3, 4.$ FEDERICO SÁNCHEZ-BRINGAS UNAM - Mexico

We study geometric properties of a branched immersion of a surface into \mathbb{R}^n , n = 3, 4 only depending on the Gauss data of the Enneper-Weierstrass representation. We analyze the branched points, and the asymptotic and curvature lines. We show models illustrating the behavior of the geometric objects under analysis, including the second order invariants of G^* -deformable branched immersions and their G^* -deformations. Namely, families of immersions into \mathbb{R}^4 constructed as the inverse of the stereographic projection of minimal immersions into \mathbb{R}^3 . This is a joint work with Juan Monterde.

Hecke algebra and motivic Chern classes of Schubert cells

Jörg Schürmann

University of Münster - Germany

We explain in the context of complete flag varieties X=G/B the relations between motivic Chern classes of Schubert cells, duality, K-theoretical stable basis of Okounkov and convolution actions of Hecke-algebras as in the work of Ginzburg and Tanisaki . This is joint work with P. Aluffi, L. Mihalcea and C. Su.

Constructing overweight deformations using polyhedra

Bernd Schober

Leibniz Universität Hannover - Germany

I will discuss the following question: Given an irreducible affine hypersurface singularity X = V(f), is it possible to find a re-embedding into a possibly larger regular ambient space Z such that, in there, X can be considered as an overweight deformation of a (not necessarily normal) toric variety Y? When this is possible, one can deduce crucial information on X by studying Y, e.g. a resolution of singularities of Y is also one for X. I will explain the ideas how such a re-embedding can be achieved by using weighted polyhedra in particular cases. This is joint work with Hussein Mourtada.

On the topology of smooth functions near their critical values

José Seade

Universidad Nacional Autónoma de M'exico - Mexico

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Topology of Real Algebraic Sets

SELMAN AKBULUT MSU - United States of America

Hexagonal geodesic 3-webs and integrability of geodesic flow

SERGEY AGAFONOV UNESP - Brazil We show that two classical problems of differential geometry, namely, existence of hexagonal geodesic 3-web and integrability of geodesic flow, are intimately related, descuss known and new examples and establish an analogue of the celebrated Graf and Sauer Theorem for Darboux superintegrable metrics.

Degeneration of the generic symmetric matrix

ARON SIMIS ET AL UFPE - Brazil

TBA

Envelopes of families of Legendre mappings in the unit tangent bundle over the Euclidean space

MASATOMO TAKAHASHI

Muroran Institute of Technology - Japan

For families of hypersurfaces with singular points, the classical definition of the envelope is vague. In order to define an envelope for a family of hypersurfaces with singular points, we consider *r*-parameter families of frontals and of Legendre mappings in the unit tangent bundle over the Euclidean space. We define an envelope for the *r*-parameter family of Legendre mappings. Then the envelope is also a frontal. We investigate properties of the envelopes. As an application, we give a condition that the projection of a singular solution of a first order partial differential equation is an envelope.

Homogeneous binary differential equations with symmetry

Patricia Tempesta

Universidade Federal de São João del Rei - Brazil

A binary differential equation (BDE) is a implicit differential equation of the form $a(x, y)dy^2 + 2b(x, y)dxdy + c(x, y)dx^2 = 0$, where the coefficients a, b, c are smooth functions. When the coefficients functions are polynomial functions of degree n we called the BDE of homogeneous BDE. This special class of binary differential equations has the property that the symmetry group is always no trivial, and furthermore, we can see that the invariant straight lines that can occur have different behavior depending on parity of the degree of the coefficients. Indeed, if n is even the invariant straight line belongs to one foliation, whereas, if n is odd it splits into pieces on the two foliations and on the discriminant set. Co-author: M. Manoel.

Generalized Slodowy Slices and deformations of singularities

Meral Tosun

Galatasaray University - Turkey

The adjoint quotient map is the key point of the Grothendieck-Brieskorn theory. When our Lie algebra is of ADE type, the adjoint orbit of codimension two in the nilpotent variety of the Lie algebra has a rational double singularity. Slodowy proved this fact in a different way and introduced an important tool, Slodowy slices, by which we can define the singularities of the adjoint orbits in the Lie algebra purely by using the Lie algebra. An exceptional choice of slices in a 6-dimesional Lie algebra will be defined to construct a family of singularities and their deformations.

A new singularity in the multiplication of polynomials

ENRIQUE VEGA CASTILLO

Universidad Nacional Auonoma de Mexico (UNAM) - Mexico

The multiplication of monic polynomials of degree n and m defines a mapping $\mathbb{R}^{n+m} \to \mathbb{R}^{n+m}$. The singularities of this mapping at a point (P,Q) have been studied by M.Chaperon and S. López de Medrano, and depend on the mcd(P,Q). In this talk we will give the general idea and describe the singularity that appears in a case not studied before. (Joint work with Santiago López de Medrano).

Fibrations and global injectivity of local homeomorphisms

JEAN VENATO SANTOS Universidade Federal de Uberlândia - Brazil

Given a path connected space X and a local fibration on its image $g: X \to I\!\!R$, we prove that g satisfies a kind of deformation and consequently we obtain the path connectedness of its level sets. Then we provide global injectivity and invertibility theorems for local homeomorphisms $f: X \to I\!\!R^n$ and we show how our results generalize some known analytical results.

Motivic zeta functions, orbifold motivic measures and Q-resolutions of singularities

Juan Viu-Sos

ICMC-USP - Brazil

The motivic zeta function $Z_{mot}(f;s)$ is a geometrical invariant associated to a complex polynomial $f \in \mathbb{C}[x_1, \ldots, x_n]$, introduced by Dener and Loeser in 1998 as a generalization of the topological zeta function $Z_{top}(f;s)$ and the *Igusa's p-adic zeta function* of f by using Kontsevich's motivic integration theory. The previous functions are classically computed in terms of an embedded resolution of singularities of $f^{-1}(0) \subset \mathbb{A}^n_{\mathbb{C}}$, where every exceptional divisor gives a "pole candidate" s_0 for $Z_{mot}(f;s)$ (or $Z_{top}(f;s)$), which could be not a real pole when one gets the final expression.

The Monodromy Conjecture affirms that any pole s_0 gives an eigenvalue $\exp(2\pi s_0)$ of the monodromy on the cohomology of the Milnor fiber of $f^{-1}(0)$.

The latter is proved in some particular cases, but one of the main difficulties to approach this conjecture is the fact that minimal resolutions of singularities does not exist for n > 2, the resolution models are complicated to compute and could become very complexes in terms of number of exceptional divisors and relations between them, providing a lot of "bad pole candidates".

In this work, we study the motivic zeta function $Z_{mot}(f;s)$ (and its specialization in $Z_{top}(f;s)$) in terms of the so-called *embedded* Q-resolutions of singularities of $f^{-1}(0)$, which are roughly embedded resolutions $\pi : X \to \mathbb{C}^n$ where the ambient space X is allowed to contain abelian quotient singularities, providing a "simpler" model with less exceptional divisors and thus less "bad pole candidates" for $Z_{mot}(f;s)$.

Centers and isochronous centers for polynomial differential systems

WILKER FERNANDES

Universidade Federal de São João del Rei - Brazil

In this investigation we study two classical problems in qualitative theory of ordinary differential equations, the center-focus and isochronicity problems. Using computational algebra tools we find conditions for the existence of two simultaneous centers for a family of planar differential systems possessing symmetry. Moreover, we investigate conditions for the isochronicity of centers in such systems and we study its global behaviour in the Poincaré disk.

The Milnor 7-sphere does not admit a special generic map into \mathbb{R}^3 DOMINIK WRAZIDLO

Kyushu University - Japan

We will present recent progress in the following problem raised by Osamu Saeki in 1993. Determine the set of integers p for which a given homotopy sphere admits a special generic map into R^p . Here, a so-called special generic map is by definition a map between smooth

map into R^p . Here, a so-called special generic map is by definition a map between manifolds all of whose singularities are definite fold points.

By means of the technique of Stein factorization we introduce and study certain special generic maps of homotopy spheres into Euclidean spaces which we call standard. Modifying a construction due to Weiss, we show that standard special generic maps give naturally rise to a filtration of the group of homotopy spheres by subgroups that is strongly related to the Gromoll filtration. Finally, we apply our result to some concrete homotopy spheres, which in particular answers SaekiÃćaĆňaĎćs problem for the Milnor 7-sphere.

Apparent contour of stable maps of surfaces with boundary

TAKAHIRO YAMAMOTO

Tokyo Gakugei University - Japan

Let M be a surface with boundary and $f_0: M \to \mathbb{R}^2$ a C^{∞} map. In this talk, we introduce the notion of an admissible minimal contour of f_0 and study admissible minimal contours of C^{∞} maps $f_0: \Sigma_{q,k} \to \mathbb{R}^2$ of compact orientable surfaces with boundary.

Bouquet decomposition for determinantal Milnor fibers

MATTHIAS ZACH

Johannes Gutenberg Universitaet Mainz - Germany

Essentially isolated determinantal singularities (EIDS) can be seen as a generalization of isolated complete intersection singularities (ICIS). They enjoy a particularly well behaved deformation theory, which allows the definition of the determinantal Milnor fiber - the generic object to which a given EIDS deforms. We will describe a bouquet decomposition for this space in terms of (suspensions of) hyperplane sections of the generic determinantal variety. Time permitting, we will discuss what is known about the these building blocks of the decomposition.