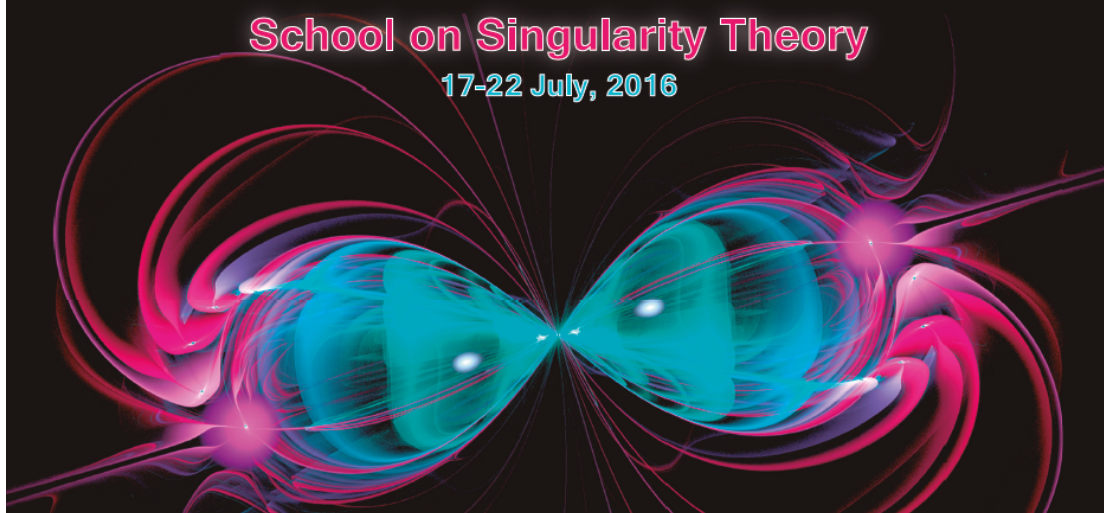# 14º INTERNATIONAL WORKSHOP ON REAL & COMPLEX SINGULARITIES
ICMC, SÃO CARLOS, BRAZIL / 24-30 JULY, 2016
School on Singularity Theory
17-22 July, 2016

BOOK OF ABSTRACTS

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Talks

Generic singularities of planar flat 3-webs

Sergey Agafonov

UNESP-IBILCE - Brazil

We propose a definition of genericity for singular flat planar 3-webs formed by integral curves of implicit ODEs and give a classification of generic singularities of such webs.

Mixed multiplicities, bi-Lipschitz equivalence of ideals and log-canonical threshold

Carles Bivià-Ausina

Universitat Politècnica de València - Spain

We introduce the notion of bi-Lipschitz equivalence of ideals and derive numerical invariants for such equivalence. In particular, we show that the log canonical threshold of ideals is a bi-Lipschitz invariant. We apply our method to several deformations $f_t : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ and show that they are not bi-Lipschitz trivial, specially focusing on the known examples of non μ^*-constant deformations. We also show a characterization of the ideals I of $\mathcal{O}_n$ of finite colength whose integral closure is equal to the integral closure of an ideal generated by pure monomials. This characterization, which is motivated by an inequality proven by Demailly and Pham, is given in terms of the log canonical threshold of I and the sequence of mixed multiplicities of I.

Co-authors: Toshizumi Fukui (Saitama University).

A Jacobian module for disentanglements and applications to Mond's conjecture

Javier Fernández De Bobadilla

BCAM/IKERBASQUE - Spain

Given a germ of holomorphic map f from $\boldsymbol{C}^n$ to $\boldsymbol{C}^{n+1}$, we define a module $M(f)$ whose dimension over $\boldsymbol{C}$ is an upper bound for the A-codimension of f, with equality if f is weighted homogeneous. We also define a relative version $My(F)$ of the module, for unfoldings F of f. The main result is that if $(n, n+1)$ are nice dimensions, then the dimension of $M(f)$ over $\boldsymbol{C}$ is an upper bound of the image Milnor number of f, with equality if and only if the relative module $My(F)$ is Cohen-Macaulay for some stable unfolding F. In particular, if $My(F)$ is Cohen-Macaulay, then we have Mond's conjecture

for f. Furthermore, if f is quasi-homogeneous, then Mond's conjecture for f is equivalent to the fact that $My(F)$ is Cohen-Macaulay. Finally, we observe that to prove Mond's conjecture, it suffices to prove it in a suitable family of examples.

Co-authors: J. J. Nuño-Ballesteros and G. Peñafort-Sanchis.

More about coincidences

JEAN-PAUL BRASSELET

CNRS Marseille - France

The Lefschetz coincidence Theorem is a generalisation of the Lefschetz fixed points Theorem. Coincidence points of two maps $f, g : X \to Y$ are points $x \in X$ such that $f(x) = g(x)$. The theory firstly provided different definitions of a Lefschetz coincidence number for compact oriented manifolds of the same dimension. On the one hand, the Lefschetz coincidence Theorem has been generalized in the case of compact manifolds of different dimensions (in the case one has a Lefschetz coincidence class) and in the case of several maps $f_1, \ldots, f_k$. On the other hand, Goresky and MacPherson generalized the Lefschetz fixed point Theorem in the case of singular varieties.

In the lecture, I will provide results obtained on the one side with Alice Libardi, Eliris Rizziolli and Thais Monis from UNESP, Rio Claro, and Marcelo Saia from USP, São Carlos, and on the other side with Tatsuo Suwa from Hokkaido University, Japan. The results concern a Lefschetz coincidence Theorem for singular varieties and local calculus of the Lefschetz coincidence class.

Closed orbits, flags and integrability for singularities of complex vector fields in dimension three

LEONARDO MEIRELES CÂMARA

Universidade Federal do Espírito Santo - Brazil

This talk is about integrability of complex vector fields in dimension three in a neighborhood of a singular point. More precisely, we study the existence of holomorphic first integrals for isolated singularities of holomorphic vector fields in complex dimension three, pursuing the discussion started in [1]. Under generic conditions, we prove a topological criteria for the existence of a holomorphic first integral. Our result may be seen as a kind of Reeb stability result for the framework of vector fields singularities in complex dimension three. As a consequence, we prove that (for the class of singularities we consider) the existence of a holomorphic first integral is invariant under topological equivalence. If we have enough time, in the final part of the talk we give a a simple proof of a finiteness criteria for groups of analytic diffeomorphisms, stated in [2].

Co-author: B. Scárdua.

References:

[1] L. Câmara & B. Scárdua, *On the integrability of holomorphic vector fields*, Discrete and Cont. Dyn. Syst. 25, Number 2, October 2009, pp. 1–13.

[2] F.-E. B. Martinez. *Groups of germs of analytic diffeomorphisms in* $(\mathbb{C}^2, 0)$. Journal of Dynamical and Control Systems, Vol. 9, No. 1, 2003, 1-32.

Affine focal set of codimension 2 submanifolds contained in hypersurfaces

MARCOS CRAIZER

Catholic University- Rio de Janeiro - Brazil

The affine focal set of a codimension 2 submanifold contained in a hypersurface of the $(n + 2)$-space is the bifurcation set of the affine distance function. For semi-umbilic immersions, the affine focal set at a point consists of n lines, while for umbilic immersions it reduces to a single line. We describe in detail the affine focal sets for some particular classes of immersions: Submanifolds contained in hyperplanes, visual contours and submanifolds contained in hyperquadrics. For the case of curves contained in surfaces of the 3-space, we classify the stable singularities. In the general case, we give a geometric characterization of the umbilic immersions in terms of the co-normal immersion of a hypersurface of the affine $(n + 1)$-space.

Topology of exceptional orbit hypersurfaces of prehomogenenous spaces

JAMES DAMON

Univ. of North Carolina - United States of America

We consider the topology for a class of hypersurfaces with highly nonisolated singularites which arise as "exceptional orbit varieties" of a special class of prehomogeneous vector spaces, which are representations of linear algebraic groups with open orbits. These hypersurface singularities include both determinantal hypersurfaces and linear free (and free*) divisors. Although these hypersurfaces have highly nonisolated singularities, we determine the topology of their Milnor fibers, complements and links. We do so by using the action of linear algebraic groups beginning with the complement, instead of using Morse-type arguments on the Milnor fibers. This includes replacing the local Milnor fiber by a global Milnor fiber which has a "complex geometry" resulting from a transitive action of an appropriate algebraic group, yielding a compact "model submanifold" for the homotopy type of the Milnor fiber. Unlike isolated singularities, the cohomology of the Milnor fibers and complements are isomorphic as algebras to exterior algebras or for one family, modules over exterior algebras; and cohomology of the link is, as a vector space, a truncated and shifted exterior algebra, for which the cohomology product structure is essentially trivial. We also deduce from Bott's periodicity theorem, the homotopy groups of the Milnor fibers for determinantal hypersurfaces in the "stable range".

The geometry of the Wigner caustic and affine equidistants of planar curves

WOJCIECH DOMITRZ

Warsaw University of Technology - Poland

We study global properties of the Wigner caustic and affine equidistants of parameterized closed planar curves. We find new results about their geometry, curvature and singular points. In particular, we consider these objects for regular closed parameterized curves with non-vanishing curvature. We present an algorithm to describe smooth branches of the Wigner caustic and affine equidistants of parameterized planar curves. By this algorithm we can find the number of smooth branches, the rotation number, the number of inflexion points and the parity of the number of cusp singularities of each branch, in particular we also study the global properties of the Wigner caustic on shell (the branch of the Wigner caustic connecting two inflexion points of a curve).

Co-author: M. Zwierzynski.

Globally subanalytic constant mean curvature surfaces in $\mathbb{R}^3$

ALEXANDRE FERNANDES

Universidade Federal do Ceará - Brazil

We prove that globally subanalytic nonsingular Constant Mean Curvature surfaces of $\mathbb{R}^3$ are only planes, round spheres or right circular cylinders.
Co-authors: L. Birbrair, L. Barbosa and M do Carmo.

Algorithms for determinantal singularities

ANNE FRÜHBIS-KRÜGER

Leibniz Universität Hannover - Germany

Determinantal singularities form a class of singularities which are at first glance more complicated than complete intersection singularities, but still contain singularities which are from certain points of view simpler than the simplest complete intersections. Up to now the study of this class of singularities has been hindered by the failure of many standard techniques, as soon as one left the realm of complete intersections. In this talk I shall present algorithmic (and thus algebraic) tools to study the topology and deformation properties for some important subclasses of such singularities in detail.
Co-author: Matthias Zach.

Insights into equisingularity through determinantal singularities

TERENCE GAFFNEY

Northeastern University - United States of America

In the 70's, 80's and 90's a beautiful theory was created to study hypersurface and complete intersection singularities. Elements of this theory extended to more general singularities, but other parts did not. For determinantal singularities all the elements of the earlier theory extend, indicating ingredients that should be included in a general theory of equisingularity. This is a report on joint projects with Antoni Rangachev, Maria Aparecida Ruas and Nivaldo Grulha, Jr.

Extrinsic geometry of plane fields

RONALDO GARCIA

Universidade Federal de Goiás - Brazil

In this talk it will be considered the one dimensional singular foliations associated to a plane field in $\mathbb{R}^3$. The generic local behavior near the curves of singularities and closed leaves will be analyzed.

Local triviality of real polynomials

Vincent Grandjean

Universidade Federal do Ceará - Brazil

We would like to discuss the problem of the local trivilization of a real polynomial (or more generally a sufficiently differentiable tame function) nearby a regular value. We are paticularly interested in finding necssary and sufficient conditions in dimension three.

Co-author: Nicolas Dutertre.

Self-dual Wulff shapes and spherical convex bodies of constant width $\pi/2$

Huhe Han

Yokohama National University - Japan

For any Wulff shape, its dual Wulff shape is naturally defined. A self-dual Wulff shape is a Wulff shape equaling its dual Wulff shape exactly. In this talk, it is shown that a Wulff shape is self-dual if and only if the spherical convex body induced by it is of constant width $\pi/2$.

Co-author: T. Nishimura.

On the polar varieties of ruled hypersurfaces

Maria Elenice Rodrigues Hernandes

Universidade Estadual de Maringá - Brazil

Our purpose in this work is to compute the polar varieties and as a consequence the polar multiplicities of a $(n-1)$-ruled hypersurface $X \subset \mathbb{C}^{n+1}$, that is defined as follows. Let $D \subset \mathbb{C}$ be a disc centered at the origin. An $(n-1)$-ruled hypersurface in $\mathbb{C}^{n+1}$ of complex dimension n is (locally) the image of a smooth map-germ $f : (\mathbb{C}^{n-1} \times D, 0) \to (\mathbb{C}^{n+1}, 0)$ given by

$$f(x_1, \ldots, x_{n-1}, t) = \alpha_0(t) + x_1\alpha_1(t) + \cdots + x_{n-1}\alpha_{n-1}(t)$$

such that the map $\alpha_i : D \to \mathbb{C}^{n+1}$ is smooth for each $i = 0, \ldots, n-1$. If each α_i is non constant we called α_0 the base curve and $\alpha_i, \ i = 1, \ldots, n-1$ the directrices curves. We prove that the polar multiplicities at the origin of X can be calculated in terms of the multiplicities of the generating curves of X. As a consequence we obtain the Euler obstruction of X.

Co-authors: M. E. Hernandes and R. Martins.

Generalized distance-squared mappings of the plane into the plane and of $\mathbb{R}^{n+1}$ into $\mathbb{R}^{2n+1}$

Shunsuke Ichiki

Yokohama National University - Japan

We define the generalized distance-squared mappings, and we concentrate on the cases of the plane into the plane and $\mathbb{R}^{n+1}$ into $\mathbb{R}^{2n+1}$ ($n \geq 1$). We classify generalized distance-squared mappings of the two cases. The case of the plane into the plane is a joint work with T. Nishimura, R. Oset Sinha and M. A. S. Ruas. The case of $\mathbb{R}^{n+1}$ into $\mathbb{R}^{2n+1}$ ($n \geq 1$) is a joint work with T. Nishimura.

Projections of crosscaps

Yutaro Kabata

Hokkaido University - Japan

We are concerned with the local geometry of projections of crosscaps. The apparent contours (the critical value sets of the projections) of crosscaps were well studied in the previous literatures through discussions with $\mathcal{A}$-equivalence. However we loose some geometrical information of crosscaps such as behaviors of the double point curves when using just $\mathcal{A}$-equivalence.

In the present work, we consider J. West's classification of submersions $\mathcal{R}^3, 0 \to \mathcal{R}^2, 0$ by local coordinate changes where the coordinate changes of the source space preserves the standard crosscap. By using the result of West's classification, we show the complete bifurcation diagrams of the apparent contours of crosscaps with the information of the crosscap points and the double point curves.

Co-author: M. Barajas (ICMC-USP).

Functions on symplectic semi-space

Konstantinos Kourliouros

Technion Institute of Technology, Haifa - Israel

We consider here the local classification problem for functions on a symplectic semi-space, i.e. on symplectic manifold with a fixed smooth hypersurface (the boundary). In particular, we show the existence of a functional modulus for the first occuring singularities and we also give its geometric description in terms of relative de Rham cohomology of the Hamiltonian line bundles associated to the pair (hypersurface, function).

Co-author: M. Zhitomirskii.

Homogeneous polynomials in two variables: A different approach

Alberto Leon Kushner-Schnur

UNAM - Mexico

A classical problem in Invariant Theory is the classification of real binary forms of degree n, under the action of the group of isomorphisms of the plane. In this talk we give an algebraic tool that allows to calculate normal forms of some orbits as well as their stabilizators. Using this method we present a classification of the forms of degree 4 and 5.

Co-authors: R. Gomez-Macedo and E. Mayorga-Saucedo.

Concentration of curvature and profiles near an isolated singular point

Rémi Langevin

Université de Bourgogne Franche Comté - France

skip Concentrations of curvature of the levels $f = \lambda$ of a complex polynomial $f : \mathbb{C}^n \mapsto \mathbb{C}$ were observed first in the late 70's; then the amount of total curvature of $f = \lambda$ contained in a neighborhood of the singular point was related to the Milnor invariants μ^i. Its more precise localization on $f = \lambda$ was studied in the 80's.

For example, the level $f = \lambda$ of the complex polynomial $f(x, y) = x^3 - y^2$ has, in a small ball B_ϵ of radius ϵ centered at the origin, total curvature close to $3 \cdot$ (universal constant). Moreover this total curvature

concentrates in neighborhoods of the three point $\sqrt[3]{\lambda}$ where, after renormalization, the level $f = \lambda$ looks like a parabola, a profile of the curve in that case.

We will show that a similar phenomenon, the apparition of a profile, occurs when studying singular curves with a singular tangent cone, and, quite often when studying surfaces with a singular point and, at that point, a singular tangent cone.

Co-author: J. C. Sifre.

On the Lê-Milnor fibration for real analytic maps

Aurélio Menegon Neto

Universidade Federal da Paraíba - Brazil

We will talk about the fibration theorems for real analytic map-germs. We will briefly discuss a new result, in collaboration with José Seade, which says that a real analytic map-germ $f : (I\!\!R^m, 0) \to (I\!\!R^n, 0)$, with $1 < n < m$ and with an isolated critical value, restricts to a locally trivial fibration: $f_| : f^{-1}(B_\eta^n\{0\}) \cap B_\epsilon^m \to B_\eta^n\{0\} \cap Im(f)$ if and only if the Euler characteristic of the intersection $f^{-1}(t) \cap S_\epsilon$ is constant for $t \in I\!\!R^n\{0\} \cap Im(f)$ sufficiently close to the origin.

Stability of degree with respect to sublevel sets

Maria Michalska

ICMC - USP - Brazil

Consider a set $S \subset \mathbb{R}^n$ and a real polynomial f. We will study behavior of f with respect to the set S. One of interesting characteristics is the degree with respect to the set. It can be defined as minimal d such that the inequality $f < g$ holds on S for some polynomial g of degree d.

First, we show that for semialgebraic S we can find a family of curves such that the degree of f w.r.t. S is given as the degree of f on a generic member of this family. Using this and properties of resolution of singularities, we are able to show stability of this degree on sublevel sets. More precisely, if $S_c = \{g < c\}$, there exist a finite set V such that if $[a, b] \cap V$ is empty, then the degree w.r.t. S_a is always equal to the degree w.r.t. S_b.

These results are connected to geometry of mappings, study of moment problems and Positivstellensaetze.

Co-author: V. Grandjean.

Homology groups of the multiple point spaces of the disentanglement of a map-germ

David Mond

University of Warwick - United Kingdom

We study the homology which appears in the images and discriminants of stable perturbations of singular map-germs. Even when the rank of this appearing homology is just 1, there are many non-isomorphic cases, as witness the three Reidemeister moves of knot theory. A better description comes from the homology of the multiple point spaces. We calculate the homology of the multiple point spaces of stable perturbations in a number of examples, including some where the germ perturbed is of corank 2. We use a variety of techniques, principally the image-computing spectral sequence and a theorem of Theo de Jong on the virtual number of D_∞ points. The talk is illustrated with many pictures. It is based on joint work with Isaac Bird.

Normally embedded surface singularities

WALTER NEUMANN

Columbia University - United States of America

Joint work with Helge M. Pedersen and Anne Pichon. A complex germ in C^n is "normally embedded" if its inner and outer geometries are bilipschitz equivalent. Among rational surface singularities the normally embedded ones are exactly the minimal singularities. We will outline the proof and describe some implications.

Classification of singular levels and Morse Bott integrable systems on surfaces

REGILENE OLIVEIRA

ICMC-USP-Brazil

Many researchers have been discussed about topological invariants to obtain the classification of systems and foliations on surfaces, in particular, the relationship between them and topology. In the classification of Hamiltonian systems with one degree of freedom on a two-dimensional compact surface Σ, the description of Liouville foliations with a Morse Hamiltonian f, is reduces to a classification of so-called atoms and molecules, as showned by Bolsinov in 1997. The tool used is the Reeb's graph of the Hamiltonian f. However, there exists integrable systems on surfaces with a first integral that is a Morse Bott function and may be with a finite number of level sets filled by equilibrium points of the system. The Reeb's graph of the first integral is insufficient in order to classifify such systems. This type of systems we called Morse Bott systems and they are a natural generalization of the Hamiltonian systems.

In this talk we present the classification, up to homeomorphisms, of closed curves and eights of saddle points on orientable closed surfaces and applied it to Morse Bott foliations and Morse Bott integrable systems to define a complete invariant for these kind of systems. We also state a realization Theorem based in two transformations and one generator (the foliation of the sphere with two centers).

Co-authors: I.S. M. Sarmiento and J. Martínez-Alfaro.

On polynomial submersions of degree 4 and the real Jacobian conjecture in $I\!\!R^2$

BRUNA ORÉFICE-OKAMOTO

UFSCar - Brazil

We prove the following version of the real Jacobian conjecture: "Let $F = (p, q) : I\!\!R^2 \to I\!\!R^2$ be a polynomial map with nowhere zero Jacobian determinant. If the degree of p is less than or equal to 4, then F is injective".

Co-author: F. Braun

Lipschitz normal embeddings and determinantal singularities

HELGE M. PEDERSEN

ICMC - USP - Brazil

An algebraic singularity $(X, 0)$ has two natural metrics. Both are defined using an embedding to Euclidian space, but are independent of the embedding up to bilipschitz equivalence. The first is the outer metric d_{out} given by restricting the Euclidian metric to $(X, 0)$. The other is the inner metric d_{in}, where the distance between two points are defined as the infimum of the lengths of curves in $(X, 0)$ between the points. It is clear that the inner distance between two points is equal or larger that their outer distance. The other way is in general not true, and one says that $(X, 0)$ is Lipschitz normally embedded if there exist a constant K, such that $d_{in}(x, y) \leq K d_{out}(x, y)$ for all $x, y \in X$. In this talk we will discuss the case of determinantal singularities. We will show that the model (or generic) determinantal singularity, that is the set of matrices

of rank less than a given number, is Lipschitz normally embedded. We will also discuss the case of a general determinantal singularity give some conditions for wich it is Lipschitz normally embedded, and examples of some there are not Lipschitz normally embedded.

First steps of local metric theory of parametrized surfaces.

Rodrigo Mendes Pereira

Universidade Federal do Ceará - Brazil

We present a criterion of normally embedded for surfaces in $I\!R^n$ from the thin zones determined by their tangent cone. For n=4, there is a close relationship with metric theory of knots. The knot themself are realized as Fukuda links of these parametrizations. An important question is if their immage is metrically un-knotted, i.e Normally embedded.

Co-authors: L. Birbrair and J.J. Nuño-Ballesteros.

Zariski - Van Kampen theorems for singular varieties

Peter Petrov

Universidade Federal Fluminense - Brazil

The classical Zariski - Van Kampen theorem gives a presentation by generators and relations of the fundamental group of the complement of an algebraic curve in the projective plane. In this talk are reviewed some generalizations from the last years, including a more general case for (possibly singular) quasi projective varieties.

Co-author: C. Eyral.

Complete classification of outer Lipschitz geometry of normal complex surfaces

Anne Pichon

Aix Marseille University - France

I will present a joint work with Walter Neumann. We give a complete classification of the outer Lipschitz geometry of a normal complex surface. It is based on a canonical decomposition of the surface as the union of hornical germs and a set of discrete invariants measuring distance inside and between them.

Equidistants and their duals for families of plane curves

Graham Reeve

University of Liverpool - United Kingdom

A generic smooth, closed plane curve will not possess two inflexion points at which the tangents are parallel, but a generic 1-parameter family of plane curves can be expected to contain isolated members with this property. In this talk I shall discuss my recent work with Peter Giblin on the generic singularities of the evolution of equidistants that occur in this situation; the most degenerate case being the 'gull singularity'. These are studied using unfoldings of singularities of maps from the plane to the plane, and more information about the overall structure, such as the occurrence of inflexions, is obtained by considering the dual structure of the equidistants.

The Nash blowing-up for curves

Faustino Agustín Romano-Velazquez

UNAM - Mexico

In this talk we are going to see the relation between the blowing-up and the Nash blowing-up for curves. We are going to establish some resemblance between both blow-ups and we are going to study the Nash resolution for curves using a more geometrical approach.

Invariants of cuspidal edges and flat surfaces

Kentaro Saji

Kobe University - Japan

In this talk, I consider developable surfaces along the singular set of a cuspidal edge which are considered to be flat approximations of the cuspidal edge. In general, developable surfaces have singularities and properties of the flat approximations are related to differential geometric properties of the original surface. In this talk, after introducing basic invariants of cuspidal edges, relations between invariants of cuspidal edges and singularities of such developable surfaces will be given. For the study of singularities, we introduce the notion of Darboux frames along cuspidal edges. Then one can obtain the structure equation and the invariants. It can be seen that these invariants are equal to the invariants which are known as basic invariants of the cuspidal edge.

Lipschitz Regularity and multiplicity of analytic sets

J. Edson Sampaio

Universiade Federal do Ceara - Brazil

We show that any Lipschitz regular complex analytic set, i.e any complex analytic set which is locally bi-lipschitz homeomorphic to an Euclidean ball must be smooth. We give partial answers to a metric version of Zariski's multiplicity conjecture. In particular, we prove the multiplicity of complex analytic surface (not necessarily isolated) singularities in C^3 is a bi-Lipschitz invariant.

Extrinsic Geometry of Surfaces in $\mathbb{R}^5$

Federico Sanchez-Bringas

UNAM - Mexico

In this talk we will present some properties of the extrisic geometry on surfaces immersed in $\mathbb{R}^5$. Our approach uses the second order invariants of the immersion and the Gauss map of the surface into the Grassmannian of two planes in $\mathbb{R}^5$. More precisely, we will analize a fiber bundle, called Osculator bundle, which provides interesting information related with asymptotic lines and some generalisations of curvature lines in this setting.
Co-authors: P. Bayard and F. Mendez.

Reflection maps

Guillermo Peñafort Sanchis

Instituto Nacional de Matemática Pura e Aplicada - Brazil

A reflection map $f \colon X \to \boldsymbol{C}^p$ is the composition of an embedding $h \colon X \hookrightarrow \mathbb{C}^p$ with the orbit map $\omega \colon \mathbb{C}^p \to \mathbb{C}^p$ of a reflection group G. While the study of general maps with corank ≥ 2 points is a hard task, reflection maps can be easily described in terms of the relation between the embedding h and the group G.

In this talk we will (a) show that a generalization of Lê's conjecture, regarding the maximum corank of injective maps, holds for reflection maps and is sharp, (b) characterize normal crossings of reflection maps, from which $\mathcal{A}$-stability obstructions follow, and (c) introduce a criterion of $\mathcal{A}$-finite determinacy for map germs of arbitrary corank in terms of multiple-point schemes, and apply it to reflection maps.

Invariants of isolated complex surface singularities

José Seade

Universidad Nacional Autonoma de Mexico - Mexico

If (V, p) is a normal 2-dimensional complex singularity, then its link M is a compact oriented 3-manifold. In this talk we will discuss topological invariants associated to the germ (V, p) and the link M.

Singularities and Dynamics of Differential Systems with Impasse Points

Jorge Sotomayor

IME - USP - Brazil

A study describing the dynamics of the solutions near points where $\det(A(x)) = 0$ (Impasse Singular Points) of systems of the form $A(x)x' = F(x)$ (Constrained Differential Systems), where A is a field of square matrices and F is a vector field in R^n will be proposed. Genericity, Stability and Bifurcations issues will be focused in the talk.

Co-authors: R. Garcia and M. Zhitomirskii.

Singularities of three functions and the product maps

Kazuto Takao

Kyoto University - Japan

I study the relationship among singularities of maps $f_1 : \mathbb{R}^n \to \mathbb{R}$ and $(f_1, f_2) : \mathbb{R}^n \to \mathbb{R}^2$ and $(f_1, f_2, f_3) : \mathbb{R}^n \to \mathbb{R}^3$. Suppose that $n \geq 3$ and (f_1, f_2, f_3) has a stable singularity at $o \in \mathbb{R}^n$. Note that the discriminant set $D_{(f_1, f_2, f_3)}$ is a regular surface, or has a cuspidal edge, or has a swallow-tail near $(f_1, f_2, f_3)(o)$ in $\mathbb{R}^3$. In this talk, I give the necessary and sufficient conditions for $D_{(f_1, f_2, f_3)}$ so that f_1 and (f_1, f_2) have stable singularities at o.

An algorithm to stratify the asymptotic set associated to a polynomial mapping from $\mathbb{C}^n$ to $\mathbb{C}^n$

Nguyen Thi Bich Thuy

IBILCE - UNESP - Brazil

Let $F : \boldsymbol{C}^n \to \boldsymbol{C}^n$ be a dominant polynomial mapping. The asymtotic set S_F of F is the set of points at which F is not proper. In other words, if a point a belongs to the asymptotic set S_F, then there exists a sequence x_k tending to infinity in the source space such that its image tends to a. It is well-known, by Jelonek, that S_F is a $(n-1)$-dimensional variety in the target space $\boldsymbol{C}^n$.

With each point a belongging to S_F, we define a 'façon" ("way") of a in terms of the way tending to infinity of a sequence corresponding. We use the idea of "façon" to give an algorithm to stratify the asymptotic of the polynomial mappings $F : \boldsymbol{C}^3 \to \boldsymbol{C}^3$ of degree 2. This algorithm can be generalized for the general case of the dominant polynomial mappings $F : \boldsymbol{C}^n \to \boldsymbol{C}^n$ of degree d.

On the smooth Whitney fibering conjecture

David Trotman

Aix-Marseille - France

The analytic Whitney fibering conjecture was recently proved by Adam Parusinski and Laurentiu Paunescu. We consider an analogue for smooth stratified sets which have a Bekka (c)-regular stratification. For each stratum of depth one there is a local foliated structure with continuous tangent planes, thus giving a smooth form of the Whitney fibering conjecture. Further there is a regular open book structure with each stratum locally a spine.

Co-authors: C. Murolo and A. du Plessis.

Unusual vanishing cycles of Matrix Singularities

Matthias Zach

Leibniz Universitaet Hannover - Germany

Isolated Cohen-Macaulay codimension 2 threefold singularities have exhibited different behavior from what could be observed in the hypersurface or complete intersection case. Namely the second Betti number of smoothings can be nonzero. We will describe a characterization of these special vanishing cycles in the case of Cohen-Macaulay type 2. This generalizes parts of the results from the preprint with Anne Fruehbis-Krueger arXiv :1501.01915.

Posters

Algorithms for computing invariants and equivariants

PATRÍCIA HERNANDES BAPTISTELLI

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Computational invariant theory may be used to realize algorithms for local or global analysis of dynamical systems with symmetry. The main point is that symmetries go along with algebraic structures such as groups, invariant rings and algebras, which are the objects of symbolic computation. In the study of equivariant dynamical systems the investigations are started by finding the general form of the symmetric vector field. In this case, each vector field can be written as a combination of fundamental invariants and equivariants, which can be computed by algorithms in a systematic way. The symbolic algorithms are also applied in the study of vector fields with other additional structures and in different contexts, such as reversible and relative equivariant vector fields. In this work we will present two algorithms in invariant theory for computing generators of the relative invariants and equivariants for arbitrary compact Lie groups G. We deal with the case when the subgroup H of symmetries is normal in G with index m greater or equal to 2. In both algorithms the knowledge of the relative Reynolds operators defined on H-invariants or on H-equivariants is shown to be an essential tool to obtain the invariants and equivariants under the whole group G.

Co-author: M. Manoel.

The semiring associated to a plane analytic curve

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Let C be an analytic plane curve given by $f = 0$ with $f = \prod_{i=1}^{r} f_i \in \mathbb{C}\{X, Y\}$ and f_i irreducible. If $\mathcal{O}$ denotes the local ring of C, that is, $\mathcal{O} = \mathbb{C}\{X, Y\}/\langle f \rangle$ then it is a known fact that the set

$$\Gamma = \left\{ \left(\dim_{\mathbb{C}} \frac{\mathbb{C}\{X, Y\}}{\langle f_1, g \rangle}, ..., \dim_{\mathbb{C}} \frac{\mathbb{C}\{X, Y\}}{\langle f_r, g \rangle} \right) \ , \ g \in \mathbb{C}\{X, Y\} \right\}$$

is a semigroup of $(\mathbb{N} \cup \{\infty\})^r$ but, in general, is not finitely generated. In this poster we introduce the concept of Standard Bases for $\mathcal{O}$ that allows to conclude that Γ is a finitely generated semiring.

Folding maps on a crosscap

MARTÍN BARAJAS S.

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We study the singularities of a family of folding maps on a crosscap. We give a list of singularities are appearing for a generic crosscap, and found geometric informations for some germs of the list.

Extending affine transverse structures with poles

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We study holomorphic foliations with an affine homogeneus transverse structure and give a characterization of the case of transversely affine foliations in terms of matrix. The work Scárdua B., the extension Lemma for the case of arbitrary codimension foliation, the author to prove the following result, generalizing the obtained in Introdução á Teoria das Folheações Algébricas Complexas.

Theorem 0.0.1. *(Extension Lemma) Let $\mathcal{F}$ is a codimension-q singular foliation on M, Λ is an analytic invariant irreducible subvariety of codimension q. Suppose:*

1. *$sing(\mathcal{F}) \cap \Lambda$ is nonempty consists of type I and type II generic singularities.*

2. *There exists a differential 1-form η defined in some neighborhood V de Λ minus Λ and its local separatrices which defines a transverse affine structure for $\mathcal{F}$ in this set $V \setminus (\Lambda \cup sep(\Lambda))$.*

Then η extends meromorphically to a neighborhood of Λ as an adapted form to Ω along Λ.

Co-author: B. Scardua.

On vertices and inflections of plane curves

Fabio Scalco Dias

UNIFEI - Brazil

We count the number of inflections and vertices concentrated at a singularity of a plane curve and obtain a relation between them.
Co-author: Farid Tari.

Salkowski curves: A family of curves with constant curvature and non-constant torsion

Marco Antônio do Couto Fernandes

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We present in this poster a summary of some works involving the study on Salkowski curves. More specifically, J. Monterde (2009) obtained a geometric characterization of such curves, namely; A curve α with constant curvature $\kappa = 1$ is Salkowski if and only if the normal vectors of the curve α make a constant angle with a fixed line in the space. Later S. Saracoglu and Y. Yayli (2014) showed that a curve α with $\kappa = 1$ is Salkowski if and only if $\det(\alpha^{(3)}, \alpha^{(4)}, \alpha^{(5)}) = 0$.

On the Plateau-Bézier Problem

Victor Hugo Caldeira Jorge Fialho

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Abstract: This reaserch consists on studying some Plateau-Bézier surfaces which are useful on engineerings and architecture. The Plateau-Bézier problem will be used to investigate mathamatics models that rule stability phenomena on geometric structures of objects produced under engineer planning, such as civil engineering

structures and mobile vehicles, for exemple. Moreover, this theory also allows the study of softening singularities and failures on objects.

On the endomorphism ring and Cohen—Macaulayness of local cohomology defined by a pair of ideals

Thiago Freitas

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In this work, we give a generalized version of Local Duality Theorem using the local cohomology defined by a pair of ideals. We investigate the behavior of the endomorphism rings $H^t_{I,J}(M)$ and $D(H^t_{I,J}(M))$ where t is the smallest integer such that the local cohomology with respect to a pair of ideals is non-zero and $D(-) := \mathrm{Hom}_R(-, E_R(k))$ is the Matlis dual functor. Also, we discuss the Cohen-Macaulayness of the Matlis dual of certain local cohomology modules with respect to a pair of ideals.

A new gap result on the 4-dimensional sphere

Rosivaldo Antonio Gonçalves

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In this note, we prove that there exists a universal number i_0 such that any metric g on the $4-$dimensional sphere S^4 and satisfying $Ric_g = 3g$ and $inj_g(S^4) \geq \dfrac{\pi}{\sqrt{4}} - i_0$ is a isometric to the round metric. Also, there exists a universal $\epsilon_0 > 0$ such that any metric g on the 4-dimensional sphere S with nonnegative sectional curvature, $Ric_g = 3g$ and $\dfrac{8}{9}\pi^2 - \epsilon_0 \geq Vol(S^4, g)$ is isometric to the round metric. These results slightly improve a rigidity theorem shown in [1] by M. Gursky. Furthermore, we apply a theorem of Gursky to improve a rigidity result about static manifolds shown in [2].

References

[1] Gursky, M. - Four-manifolds with $\delta W^+ = 0$ and Einstein constant of the sphere, Math. Ann., 318 (2000), 417-431.

[2] H. Seshadri, On Einstein four-manifolds with S^1-actions, Math. Z. 247 (2004), no. 3, 487-503.

Co-authors: E. R. Barbosa and A. Freitas.

Asymptotic lines on surfaces in $\mathbb{R}^4$.

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Consider a regular surface $\mathbb{M}^2$ in $\mathbb{R}^4$. Associated to $\mathbb{M}^2$ is defined the curvature ellipse $\mathcal{E}(p)$ (see for example [1]) contained on the normal plane $N_p\mathbb{M}^2$.

At the points $p \in \mathbb{M}^2$ where the origin $(0,0)$ is contained on the exterior region delimited by the ellipse $\mathcal{E}(p)$ we have two directions tangent to the ellipse.

In a local chart (u, v), this pair of directions is defined by the following binary implicit differential equation

$$(e_1 f_2 - e_2 f_1)du^2 + (e_1 g_2 - e_2 g_1)dudv + (f_1 g_2 - f_2 g_1)dv^2 = 0,$$

where $II_1 = e_1 du^2 + 2f_1 dudv + g_1 dv^2$ and $II_2 = e_2 du^2 + 2f_2 dudv + g_2 dv^2$ are the fundamental forms associated to a orthonormal basis of $N_p\mathbb{M}^2$. The integral curves of this equation are called asymptotic lines.

In this work we study closed asymptotic lines on surfaces in $\mathbb{R}^4$.

Co-author: R. Garcia.

Reference:

[1] R.Garcia, J. Sotomayor. *Differential equations of classical geometry, a qualitative theory. Publicações Matemáticas, IMPA, 27º Colóquio Brasileiro de Matemática, 2009.*

Some results on the Hilbert's 16th Problem for two families of piecewise polynomial differential systems

Jackson Itikawa

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The second part of the famous Hilbert's 16th Problem deals with the number and location of limit cycles of a planar polynomial vector field of degree n. In this work we apply the averaging theory of first order for discontinuous differential systems to investigate the bifurcation of limit cycles in quadratic and cubic polynomial differential systems with a uniform isochronous center. In order to obtain our results, these systems are perturbed inside the classes of all discontinuous quadratic and cubic polynomials differential systems respectively, splitting the plane in four quadrants by the axes of coordinates.

Co-authors: J. Llibre (UAB, Spain), A. Mereu (UFSCar – Sorocaba) and R. Oliveira (ICMC – USP)

Invariant theory for compact subgroups of the Lorentz group acting on the Minkowski space

Leandro Nery de Oliveira

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We investigate some compact groups that are subgroups of the Lorentz group and its respective action in Minkowski's space, from the point of view of invariant theory. We established a general form of the involution of Lorentz and show examples of compact subgroups of the Lorentz group.

Co-author: M. G. Manoel.

Equiaffine Darboux Frames for Codimension 2 Submanifolds

Luis Floral Espinosa Sanches

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Consider a codimension 1 submanifold $N^n \subset M^{n+1}$, where $M^{n+1} \subset I\!\!R^{n+2}$ is a hypersurface. There are some important examples of submanifolds that admit a vector field tangent to M and transversal to N whose derivative in any direction of N is contained in N. When this is the case, one can construct transversal plane bundles and affine metrics on N with the desirable properties of being equiaffine and apolar. We study the envelope of tangent spaces of M along N and we generalizes the concept of tangent developable surface of a surface along a curve.

Co-authors: Marcos Craizer and Marcelo J. Saia.

Singular levels and topological invariants of Mors Bott foliations on non orientable surfaces

Ingrid Sofia Meza-Sarmiento

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We classify up to ambient homeomorphism simple closed curves and eights of saddle points on orientable closed surfaces. The classification obtained here is applied to Morse Bott foliations on non orientable closed surfaces to define a complete topological invariant.

Co-authors: J. Martínez-Alfaro and R. D. S. Oliveira.

The cr-invariant and the configurations of generic curves for surfaces in $\mathbb{R}^4$.

JORGE LUIZ DEOLINDO SILVA

Universidade Federal de Santa Catarina - Brazil

We study the geometry of surfaces in $\mathbb{R}^4$ associated to contact with lines. We list the possible configurations that occur on parabolic, S_2, B_2, flecnodal and asymptotic curves at $P_3(c)$-point through of the cr-invariant.

Existence of Moduli for Hölder Equivalence Analytic Functions

JOSERLAN PEROTE DA SILVA

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In this work, we show that Holder equivalence of analytic functions germs $(C^2, 0) \rightarrow (C, 0)$ admits continuous moduli. More precisely, we constructed an invariant of the Holder equivalence of such germs that varies continuously in a family $f_t : (C^2, 0) \rightarrow (C, 0)$. For a single germ f_t the invariant of ft is given in terms of the leading coeficients of the asymptotic expansion of ft along the branches of generic polar curve of ft .