BOOK OF ABSTRACTS

DAY OF YOUNG RESEARCHERS - TALKS AND POSTERS

22 July 2016

FAPESP CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico CAPES ICMC USP SÃO CARLOS

Table of Contents

Topological Classification of Map Germs

Erica Boizan Batista
UFSCAR - Brazil

The topological type of a finitely determined map germ $f : (\mathbb{R}^3, 0) \to (\mathbb{R}^2, 0)$ is given by the so-called link of f. The link of f is obtained by taking a small enough representative $f : U \subset \mathbb{R}^3 \to \mathbb{R}^2$ and the intersection of its image with a small enough sphere S^1_δ centered at the origin in $\mathbb{R}^2$. As a consequence of Fukuda's theorem, two finitely determined map germs $f, g : (\mathbb{R}^3, 0) \to (\mathbb{R}^2, 0)$ are topologically equivalent if their associated links are topologically equivalent. Inspired by the works of Arnold, Maksymenko and Prishlyak we introduce an adapted version of the Reeb graph that turns out to be a complete topological invariant for the links.

Singularities of general polynomial mappings

Michal Farnik
Jagiellonian University - Poland

I will start by describing the topology of a general polynomial mapping $F : \mathbb{C}^2 \to \mathbb{C}^2$ of degree (d_1, d_2). I will show that the set $C(F)$ of critical points of F and the discriminant $F(C(F))$ are birationally equivalent curves. Moreover, the projective closure of $C(F)$ is smooth whereas $F(C(F))$ has only cusps and nodes as singularities. I will calculate their number in terms of d_1 and d_2.

Then I will move on to general $\mathbb{C}^3 \to \mathbb{C}^3$ mappings. I will show that they have only two-folds, cusps and swallowtails as singularities and calculate the degree of their locus. I will conclude with describing possible generalizations.

Co-authors: Z. Jelonek and M.A.S. Ruas.

On the Nash modification of a germ of complex analytic singularity.

Arturo Giles Flores
Universidad Autónoma de Aguascalientes - Mexico

For a germ of analytic singularity $(X, 0) \subset (\mathbb{C}^n, 0)$ the set of limits of tangent spaces plays a big role in the study of equisingularity. This set is obtained as the preimage $\nu^{-1}(0)$ of the Nash modification $\nu : NX \to X$ and it is then a subvariety of the Grasmannian $G(d, n)$.

When X is a hypersurface the Grassmannian $G(d, n)$ is a projective space $\check{\mathbb{P}}^{n-1}$ and the set $\nu^{-1}(0)$ can be described via projective duality by a finite family of subcones of the tangent cone $C_{X,0}$.

The generalization of this result to germs of arbitrary codimension needs to replace the Nash modification NX by the conormal space $C(X)$ and one of the key points that makes this generalization work is that conormal varieties can be characterized as Legendrian subvarieties of projectivized cotangent spaces with their canonical contact structure. In this spirit we try to characterize analytic subvarieties Z of $\mathbb{C}^n \times G(d, n)$ such that:

1. Z has dimension d.
2. Its image (by the projection) X in $\mathbb{C}^n$ has dimension d.
3. Z is the Nash modification of X

On the Merle Theorem

Mauro Fernando Hernández Iglesias
UEM - Brazil

We present completely the description of the packages given in the famous Theorem by Merle for the polar of a generic element in a fixed topological class. In particular, we obtain explicitly the topology of the polar curve from the cluster as indicated by Casas Alvero. Using our analysis it is possible to characterize all semigroups for which the polar of a generic curve with genus g has only components of genus less than g.

Limit cycles in discontinouous polynomial differential systems of degree 2 and 3 with two straight lines of discontinuity

Jackson Itikawa
ICMC/ USP - Brazil

We apply the averaging theory of first order for piecewise discontinuous differential systems to investigate the bifurcation of limit cycles in the polynomial differential systems $\dot{x} = -y + x^2,\ \dot{y} = x + xy$, and $\dot{x} = -y + x^2y,\ \dot{y} = x + xy^2$, when they are perturbed inside the class of all discontinuous quadratic and cubic polynomials differential systems with four zones separated by the axes of coordinates, respectively.

Co-authors: J. Llibre, A. Mereu and R. Oliveira.

On real Weierstrass points of real algebraic curves of genus four

Cristhian Emanuel Garay Lopez
Universidade Federal Fluminense - Brazil

It is well known that any complex algebraic curve of genus four (non-hyperelliptic) has the same number of Weierstrass points (sixty). This is no longer true in the real case, in fact, we still do not know what is the maximal number of real Weierstrass points $w_R(C)$ that a real algebraic curve C of genus four (non-hyperelliptic) may have.

Our aim is to show that if C is a real algebraic curve of genus four (non-hyperelliptic) with *controlled singularities*, then

$$w_R(C) = -\chi(\nu^{-1}(C^*(R))),$$

where $\nu : X \longrightarrow C^*$ is certain resolution of singularities of the dual variety C^* of C and χ is the Euler characteristic of topological spaces.

The codimension-one discriminant of a stable mapping

Mirna Lissette Gomez-Morales

Universidad Nacional Autonoma de Mexico - Mexico

We study the $\mathcal{A}_e$-codimension one map-germs that may be obtained by restricting the target of a minimal stable map-germ $F : (\boldsymbol{C}^n, 0) \to (\boldsymbol{C}^p, 0)$. We call the set of hyperplanes $\{L : \text{codim}_{\mathcal{A}_e}(F|_L) > 1\}$ the codimension-one discriminant of the stable map F and prove that it defines an analytic variety $V(I_F)$ in the space of hyperplanes, visualised inside $\boldsymbol{C}^p$.

Moreover, we describe the codimension-one discriminant of a number of minimal stable mappings, focusing on unfoldings of map-germs from $\boldsymbol{C}^2$ to $\boldsymbol{C}^3$. Although, in principle, the codimension-one discriminant seems to be a linear free divisor, we show that this property does not hold for every minimal stable map-germ F.

Co-author: D. Mond.

The improved isoperimetric inequality and the Wigner caustic of planar ovals

Michal Zwierzynski

Warsaw University of Technology - Poland

The classical isoperimetric inequality in the Euclidean plane $\mathbb{R}^2$ states that for a simple closed curve M of the length L, enclosing a region of the area A, one gets $L^2 \geqslant 4\pi A$ and the equality holds if and only if M is a circle. We will show that if M is an oval, then $L^2 \geqslant 4\pi A + 8\pi|A_{0.5}|$, where $A_{0.5}$ is an oriented area of the Wigner caustic of M, and the equality holds if and only if M is a curve of constant width.

Characteristic p, Singularities of Hypersurfaces and the Milnor Number

João Hélder Olmedo Rodrigues

UFF - Brazil

In this presentation I will talk about some aspects of hypersurfaces singularities over fields of characteristic p. We will see how some pathologies which appear in the definition of the classical invariant introduced by Milnor are connected with the failure, in positive characteristic, of the familiar Bertini's (or Sard's) theorem. Also, we will discuss what could be the best definition of a Milnor number for a hypersurface singularity in positive characteristic.

In the special case of plane branches we are able to present a condition on the semigroup of values which is sufficient for the absence of the so called wild vanishing cycles. We conjecture that the condition is also necessary but we don't have, at present, a complete proof.

Co-authors: A. Hefez and R. Salomão.

Equivalence between the hypothesis of Milnor-Lê fibration of $f\bar{g}$ and Milnor fibration of f/g in $(\mathbb{C}^2, 0)$

Josefa Genyle do Nascimento Santana

UFF - Brazil

Let $f, g : (\mathbb{C}^n, 0) \longrightarrow (\mathbb{C}, 0)$ be holomorphic germs such that $f^{-1}(0)$ and $g^{-1}(0)$ have no common irreducible components.

The Milnor-Lê fibration theorem for $f\bar{g}$ states that if $f\bar{g} : (\mathbb{C}^n, 0) \longrightarrow (\mathbb{R}^2, 0)$ have a isolated critical value at 0, then $\psi_{f\bar{g}} : \mathbb{S}_\varepsilon \setminus \mathcal{L}_{f\bar{g}} \longrightarrow \mathbb{S}^1$ defined by $\psi_{f\bar{g}}(z) = \dfrac{f\bar{g}(z)}{|f\bar{g}(z)|}$ is a fibration of the multilink $\mathcal{L}_f \cup -\mathcal{L}_g$.

If f/g is semitame at 0, then the Milnor fibration theorem for f/g says that $\phi_{f/g} : \mathbb{S}_\varepsilon \setminus \mathcal{L}_{f/g} \longrightarrow \mathbb{S}^1$ given by $\phi_{f/g}(z) = \dfrac{f/g(z)}{|f/g(z)|}$ is a fibration with the same multilink as above.

A natural question is to compare these hypothesis. We will show that in $(\mathbb{C}^2, 0)$ they are equivalents.

This work is based in the article Fibred multilinks and singularities $f\bar{g}$ of Anne Pichon and José Seade.

A Counterexample for Kedlaya Theorem on the Algebraic Closure of $K((t))$ in Positive Characteristic

Reillon Oriel Carvalho Santos

Universidade Federal Fluminense - Brazil

In this work, we prove that given an algebraically closed field K of positive characteristic, the set of the twist-recurrent series over K forms an algebraic extension of $K((t))$. We also prove that this field is not algebraically closed. Roughly speaking, twist-recurrent series are power series with rational exponents such that the set of coefficients has subsequences satisfying recurrence relations.

Whitney equisingularity of families of surfaces

Otoniel Nogueira da Silva

Universidade de São Paulo - Brazil

In this talk, we speak about the Whitney equisingularity of families of singular surfaces in $\mathbb{C}^3$ parametrized by $\mathcal{A}$-finitely determined map germs. We show that when $f : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$ has corank 1 and is finitely determined, then every 1-parameter unfolding F of f which is topologically equisingular is also Whitney equisingular, furthermore, in this case the

Milnor number $\mu(D(f_t))$, where $D(f_t)$ is the double point curve of f_t, is the unique invariant necessary to control the Whitney equisingularity of the family f_t. We also give examples of corank 2 families f_t, we show that in some cases the topological equisingularity implies the Whitney equisingularity of the family, for instance, the class of *double fold maps* introduced by Ballesteros and Marar.

Co-author: M. A. Soares Ruas.

Bifurcation and Genericity Conditions for Axial Cycles of Surfaces Immersed in $I\!R^4$

Flausino Lucas Neves Spindola

Universidade Federal do Maranhão - Brazil

We present the description of the structure of closed principal lines of immersions of surface M^2 into $I\!R^4$, which are periodic solutions of the differential equations of lines of axial curvature. We study the perturbation of principal axial cycles, and we obtain results about genericity of hyperbolic and semi-hyperbolic principal axial cycles.

Surfaces around closed principal lines in Minkowski space $\mathbb{R}^{2,1}$

Dimas Noe Tejada Tejada

Universidade Federal de Goiás - Brazil

Given a smooth space-like closed curve with total torsion zero in Minkowski 3-space, we construct a germ of smooth surfaces that contains it as a hyperbolic principal cycle.

Co-author: R. A. Garcia.

On equivariant binary differential equations

Patrícia Tempesta

ICMC-USP - Brazil

This work introduces the study of occurrence of symmetries in binary differential equations (BDEs). These are implicit differential equations given by the zeros of a quadratic 1-form, $a(x,y)dy^2 + b(x,y)dxdy + c(x,y)dx^2 = 0$, for a, b, c smooth real functions defined on an open set of $\mathbb{R}^2$. Generically, solutions of a BDE are given as leaves of a pair of foliations, and the appropriate way to define the action of a symmetry must depend not only whether it preserves or inverts the plane orientation, but also whether it preserves or interchanges the foliations. The first main result reveals this dependence, which is given algebraically by a formula relating two group homomorphisms defined on the symmetry group of the BDE. The second main result adapts algebraic methods from invariant theory for representations of compact Lie groups on the space of quadratic forms on $\mathbb{R}^n$, $n \geq 2$. With that we obtain an algorithm to compute general forms of quadratic forms. Now, symmetric quadratic 1-forms

are in one-to-one correspondence with equivariant quadratic forms on the plane, so these are treated here as a particular case. We then apply the result to obtain the general forms of equivariant quadratic 1-forms under each compact subgroup of the orthogonal group $\mathrm{O}(2)$.

Co-author: M. Manoel.

Non-degenerate jump of Milnor numbers of surface singularities

Justyna Walewska, PhD

Faculty of Mathematics and Computer Science, University of Lodz - Poland

The jump of the Milnor number of an isolated singularity f_0 is the minimal non-zero difference between the Milnor numbers of f_0 and one of its deformations f_s. We give a formula for the jump in some class of surface singularities in the case deformations are non-degenerate.
